Stability and Capacity of Wireless Networks with Probabilistic Receptions: Part I—General Topologies

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***Abstract*— We study the stability and the capacity problems in packetized wireless networks. Communication medium is modeled using probability density functions that determine the packet reception probabilities. The model subsumes several previous models as special cases, and it is suitable for networks with time-varying topology and channels. Our main result is a characterization of the stability and the capacity regions using network flows. We also introduce a class of control policies sufficient to achieve every rate inside these regions. In Part II, we apply the proposed policies and the flow analysis to regular networks. We obtain closed-form expressions for the capacity of Manhattan networks (two-dimensional grid) and ring networks (circular array of nodes). We analyze the performance loss due to suboptimal medium access and routing. We also investigate the impact of link fading, link state information, and variable connectivity on achievable rates in Manhattan networks.**

***Index Terms*— Wireless networks, regular networks, Manhattan, multipacket reception, capacity, stability, transport capacity, slotted ALOHA, scheduling, optimal connectivity.**

I. Introduction

THE objective of this work is twofold. First, we provide a general approach to characterizing the capacity and stability regions of networks with a probabilistic reception model. This model, defined by the conditional probability of successful receptions given the subset of transmitting users, is sufficiently general to include multipacket receptions and links with ergodic fading. Second (in Part II), we aim to provide insights and design guidelines by examining the class of one-dimensional (ring) and two-dimensional (Manhattan) regular networks. Having obtained closed-form expressions for the capacity, we are able to quantify the loss incurred by suboptimal protocols, the gain obtained by using link state information, and the effects of increasing connectivity.

The network capacity problem deals with finding the fundamental limits on achievable communication rates in wireless networks. A set of rates between source-destination pairs is called achievable if there exists a network control policy that guarantee those rates. The closure of the set of achievable rates is the *capacity region* of the network. Our main result is a characterization of the capacity region using network flows. In the flow characterization one needs to assign a probability density over the set of transmission schedules for medium access (MAC). Similarly, routing amounts to assigning a probability density over the set of routes. We show that all rates inside the capacity region can be represented as a flow feasible with certain probability densities for MAC and routing. To establish this result we introduce a class of control policies that do randomized routing and medium access. These will be called *randomized time-division* (RTD) policies since their MAC can be viewed as a randomized version of TDMA (time-division multiple access). Our characterization shows that the *marginal* link success probabilities determine the capacity region, not the joint ones.

In the capacity analysis the notion of *transport capacity* plays an important role. The transport capacity introduced by Gupta and Kumar [1] measures the delivery rate times the distance packets travel. In this paper, we extend the definition of transport capacity to networks with probabilistic receptions. This extension allows us to handle the transport capacity in a more general setting where the network possibly has time variation and the distance metric is not Euclidean. The generalized transport capacity is used extensively in proving upper bounds on the capacities of regular networks.

In the capacity analysis, it is assumed that there are always packets to be delivered at the source nodes. However, in reality the data packets arrive randomly in time, and for proper network operation the node buffers should be kept stable. Intuitively, the network is called stable if the node buffers do not overflow during operation. The *stability region* is the closure of the set of arrival rates at which the network can be stabilized using a control policy. Stability depends both on the rate of packet arrivals and the rate of packet departure from the network. The latter is of course closely related to the notion of achievability. One would expect that the stability region must be inside the capacity region; this is true, since the delivery rate is equal to the arrival rate in stable networks. The converse, however, is false in general; some part of the capacity region may lie outside the stability region in networks with probabilistic receptions. In Section II we provide some mild conditions on the network under which the stability and the capacity regions are the same. In

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proving this result, we again exploit the flow characterization and the RTD policies. In particular, we show that for every arrival rate inside the stability region there exists an RTD policy stabilizing the network.

In the literature, the wireless network stability problems have been studied extensively both for networks with centralized scheduling [2]–[13] and the ALOHA protocol [14]–[19]. Our problem formulation is closest to the model used by Tassiulas and Ephremides [2] who studied the network stability with a specific probabilistic model and characterized the network stability region. They also gave an elegant throughput-optimal policy that stabilizes the network at all arrival rates in the stability region. In [2], the packet arrival process is assumed to be independent and identically distributed (i.i.d.), and the network stability is analyzed in a Markovian framework using the Lyapunov functional approach. In this paper, we consider a more general network model with stationary and ergodic (not necessarily i.i.d.) arrival processes, and our stability notion is slightly different. We also use a different methodology (a dominant system approach) in stability analysis.

The organization of the paper is as follows. In the next section the network model is introduced. In Section III, the RTD policies are introduced, and the network stability and capacity regions are characterized. Also, an upper bound on the achievable rates is developed using the transport capacity. The proofs of the main theorems are relegated to the appendix. We conclude this part in Section IV. In Part II [20], we apply the developed tools to compute the capacity of regular networks.

Sets will be denoted by script letters. For a set $\mathcal{A}$, $|\mathcal{A}|$ is the number of elements in $\mathcal{A}$, and $\mathcal{A}^n = \{(a_1, a_2, \cdots, a_n) : a_i \in \mathcal{A}, i = 1, 2, \cdots, n\}$. The set of non-negative integers is $\mathcal{Z}_+ = \{0, 1, 2, \cdots\}$.

II. Network Model

Suppose that time is divided into unit length slots, and slot $t \in \mathcal{Z}_+$ is defined as the half-open interval $[t, t+1)$. Let $\mathcal{N} = \{1, 2, \cdots, N\}$ be the set of nodes in the network, and $\mathcal{L} = \{(i, j) : i, j \in \mathcal{N}, i \neq j\}$ be the set of links. For link $l = (i, j)$, $t(l)$ denotes the transmitter node i, and $r(l)$ denotes the receiver node j.

We represent transmissions using binary vectors. A packet destined for node j will be called a j-packet. Let $E_{lj}(t)$ be equal to 1 if a j-packet is *transmitted* over link l in slot t, and 0 otherwise. Similarly, let $F_{lj}(t)$ be equal to 1 if a j-packet is *successfully received* over link l in slot t, and 0 otherwise. Over each link a single packet can be transmitted, *i.e.,* $\sum_j E_{lj}(t) \in \{0, 1\}$. Define $E(t) = (E_{lj}(t) : l \in \mathcal{L}, j \in \mathcal{N})$, and $F(t) = (F_{lj}(t) : l \in \mathcal{L}, j \in \mathcal{N})$. The set of transmissions in slot t is $\mathcal{E}(t) = \{l \in \mathcal{L} : E_{lj}(t) = 1 \text{ for some } j\}$, the set of receptions is $\mathcal{F}(t) = \{l \in \mathcal{L} : F_{lj}(t) = 1 \text{ for some } j\}$.

Time variation in the network topology and the channel qualities are modeled using *states*. Let $\mathcal{V}$ be the set of states, and $v(t) \in \mathcal{V}$ be the state of the network in slot t. The state $v(t)$ can be any network parameter affecting the receptions; examples include channel gains between users and spatial locations of the nodes. It is assumed that the process $(v(t) : t \in \mathcal{Z}_+)$ is stationary and ergodic, and the probability of state v is $p(v)$ in the stationary distribution[1].

Wireless channels in general are subject to random fading, and neighboring transmissions interfere with one another. Because of these reasons, some of the transmitted packets may not be received successfully. We model the channel characteristics and the reception errors using a conditional probability density function (pdf) π. In each slot the received packets $\mathcal{F}(t)$ are determined according to the pdf $\pi(\,\cdot\,; \mathcal{E}(t), v(t))$. The quantity $\pi(\mathcal{F}(t)\,; \mathcal{E}(t), v(t))$ is the probability that the successful reception set is $\mathcal{F}(t)$ given that the transmission set is $\mathcal{E}(t)$ and the network is in state $v(t)$. The pdf π also specifies the *transmission constraints* (such as half-duplex nodes) and the *network topology*: if a set of transmissions $\mathcal{E}$ is physically impossible, then the set of successful receptions is empty with probability 1. In multi-hop networks there may be such impossible $\mathcal{E}$, since the nodes are typically restricted to communicate with neighbors. Specific choices of π give several previous models such as the collision channel [21], [22], the MPR model [23]–[30], and others [2]–[5], [31], [32].

We assume that new packets arrive at the network randomly according to a stochastic process. Let $A_{ij}(t)$ be the number of j-packets arrived at node i in slot t. (Equivalently, we say that node i *generated* $A_{ij}(t)$ j-packets in slot t.) The arrival process $A(t) = (A_{ij}(t) : i, j \in \mathcal{N})$ is assumed to be stationary and ergodic with mean $\lambda = (\lambda_{ij} : i, j \in \mathcal{N})$. In slot $t = 0$ the network starts operation with empty queues. The nodes can store an unlimited number of packets in their buffers, and a packet does not leave the network unless it reaches its destination. At time t the number of j-packets at node i is denoted by $n_{ij}(t)$. Define $n(t) = (n_{ij}(t) : i, j \in \mathcal{N})$ and $n_i(t) = \sum_j n_{ij}(t)$. Time evolution of each queue is described by

$$n_{ij}(t+1) = \begin{cases} n_{ij}(t) - \sum_{l \in \mathcal{L}:\, t(l)=i} F_{lj}(t) \\ + \sum_{l \in \mathcal{L}:\, r(l)=i} F_{lj}(t) + A_{ij}(t), & \text{if } i \neq j \\ \\ 0, & \text{if } i = j. \end{cases} \tag{1}$$

In each slot t, a control policy determines $E(t)$. The policies we consider are causal, and they can be randomized. In slot t the policies assume the knowledge of $v(t)$.[2]

We have the following assumption, which is expected to be satisfied in practice.

(A1) Let $\mathcal{E}$ be the set of transmissions, and $\mathcal{F} \subset \mathcal{E}$ be the set of receptions. Define the marginal probability of successfully receiving $\mathcal{F}$ given $\mathcal{E}$ as

$$\Pi(\mathcal{F}; \mathcal{E}, v) = \sum_{\mathcal{F}':\, \mathcal{F} \subset \mathcal{F}'} \pi(\mathcal{F}'; \mathcal{E}, v).$$

[1] We suppose that the set $\mathcal{V}$ is countable. However, this assumption is for notational convenience. The following results hold also when $\mathcal{V}$ is uncountable.

[2] The formal definition of a policy is the following. The information available up to time $t-1$ is $I(t) := (A(r), F(r), v(r) : r = 0, 1, \cdots, t-1)$. Policies can be randomized; suppose that $U = (U(t) : t \in \mathcal{Z}_+)$ is a vector containing i.i.d. random variables which are used for randomization. A *policy* is a sequence of functions $\phi = (\phi_t : t \in \mathcal{Z}_+)$, such that $E(t) = \phi_t(I(t), v(t), U)$, *i.e.,* ϕ_t determines $E(t)$ according to $I(t), v(t), U$. The randomization variables $U(t)$, state $v(t)$ and arrival processes $A(t)$ are assumed to be *jointly* stationary and ergodic.

For all $\mathcal{E}' \supset \mathcal{E}$ and $v \in \mathcal{V}$, we require

$$\Pi(\mathcal{F};\mathcal{E},v) \geq \Pi(\mathcal{F};\mathcal{E}',v),$$

i.e., the marginal probability of success is lower when there are more transmissions.

III. Main Results

In this section, we characterize the capacity and stability regions of networks. In our characterization the so-called RTD policies play an important role. The basic idea behind the RTD policies is the assignment of random routes and the use of random schedules according to some probability distribution. In order to make these ideas precise, we need a few more definitions that are presented in the next subsection.

A. Randomized Time-Division (RTD) Policies

A *path* from node i_0 to i_k is a vector $(i_0, i_1, \cdots, i_k) \in \mathcal{N}^{k+1}$ such that $i_0, \cdots, i_k$ are different nodes. Denote the set of all paths from node i to node j by $\mathscr{P}_{ij}$, and define $\mathscr{P} = \cup_{i,j\in\mathcal{N}} \mathscr{P}_{ij}$. For some $P = (i_0, i_1, \cdots, i_k) \in \mathscr{P}$, we say that link l is in path P (*i.e.*, $l \in P$) if $(t(l), r(l)) = (i_j, i_{j+1})$ for some $j \in \{0, \cdots, k-1\}$. Let $\mathscr{E}$ denote the power set of $\mathcal{L}$. A *routing vector* is a vector $H = (x_P \geq 0 : P \in \mathscr{P})$ satisfying

$$\sum_{P\in\mathscr{P}_{ij}} x_P = 1, \text{ for all } i \neq j.$$

Similarly, a *scheduling vector* is a vector $G = (p(\mathcal{E};v) \geq 0 : v \in \mathcal{V}, \mathcal{E} \in \mathscr{E})$ satisfying

$$\sum_{\mathcal{E}\in\mathscr{E}} p(\mathcal{E};v) = 1, \text{ for all } v \in \mathcal{V}.$$

An RTD policy is specified by the vectors G, H and the arrival rate λ.[3] The vectors G and H will be viewed as probability densities over routes and transmission schedules. Three mechanisms used in an RTD policy are the following.

Routing: The packets are identified with their routes, and every packet is assigned a fixed route randomly once it is generated. If node i generates a packet for node j, route $P \in \mathscr{P}_{ij}$ is assigned with probability x_P.

Medium Access: In every slot, a randomly chosen schedule is applied. In slot t, transmission schedule $\mathcal{E}(t) = \mathcal{E}$ is chosen with probability $p(\mathcal{E}; v(t))$.

Queuing discipline: After the transmission schedule is chosen, every node chooses the types of packets it will transmit: If node i is scheduled to transmit over link l, a route P (such that $l \in P$) is chosen randomly with probability

$$Q(P,l) = \frac{x_P\lambda_{ij}}{\sum_{i,j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{ij}:\, l\in P} x_P\lambda_{ij}}. \quad (3)$$

Then, over link l a packet with route P is transmitted if node i has a packet with route P. Eqn. (3) assures fairness: each route passing through link l is allocated bandwidth proportional to its traffic rate $x_P\lambda_{ij}$.

[3] This λ should be interpreted as the *target* arrival rate (the network is expected to support) rather than the actual arrival rate.

Now specification of the RTD policies is complete. Before analyzing network stability, we will discuss some connections between RTD policies and TDMA. In ad hoc networks, TDMA can be done by applying a sequence of transmission schedules periodically (see [21], [22]). The knowledge of queue lengths is not required in TDMA. Therefore, it can be applied in a distributed network, but the cycles should be designed with a prior knowledge of the arrival rates network should support. The medium access in RTD policies can be viewed as a generalization of TDMA to networks with time-variation. This generalization is done by choosing a random transmission schedule in each slot instead of cycling through different transmission schedules.

The RTD policies can be applied in distributed networks if every node has access to the network state $v(t)$ in each slot (This is the case if the network has a single state, or cycles through states periodically, or if there is a feedback link from a central controller broadcasting the state). One possibility for distributed implementation is the use of pseudo-random number generators which were previously proposed in [33], [34]. In case all nodes use a common pseudo-randomization algorithm (or, a common seed [33]) then a pseudo-random vector $\mathcal{E}(t)$ can be picked according to distribution $p(\,\cdot\,; v(t))$ by each node locally. Once MAC is done using pseudo-randomization, routing and the queuing discipline can be readily applied distributively.

B. Stability

In this subsection, we define stability and the network flows. We then characterize the stability region.

Definition In a network with arrival rate λ and with some policy, node $i \in \mathcal{N}$ is called $stable$ if the distribution of queue length $n_i(t)$ converges to some proper distribution W as $t{\to}\infty$, *i.e.*,

$$\lim_{t\to\infty} \Pr\{n_i(t) \leq \theta\} = W(\theta) \text{ and } \lim_{\theta\to\infty} W(\theta) = 1.$$

Node $i \in \mathcal{N}$ is called $substable$ if

$$\lim_{\theta\to\infty} \liminf_{t\to\infty} \Pr\{n_i(t) \leq \theta\} = 1.$$

We call a network is stable if all nodes in the network are substable; it is called unstable otherwise.

Substability is a condition weaker than stability: a stable node is always substable, but the converse is not always true. Substability admits a heuristic interpretation. Supposing that θ is the buffer capacity of node i, we can interpret $\limsup_{t\to\infty} \Pr\{n_i(t) > \theta\}$ as the asymptotic buffer overflow probability of node i. A node is substable if and only if its asymptotic buffer overflow probability goes to zero as the buffer size θ tends to infinity. Substability is equivalent to the more common notion of tightness [35]. In the wireless networking context, as a network stability criterion, substability is first used by Tsybakov and Bakirov [14]. Depending on the network model, other stability notions are also used in the literature [15]–[19], [2]–[9].

Arrival rate $\lambda = (\lambda_{ij} : i,j \in \mathcal{N})$ is called *stabilizable* if there exists a policy that makes the network stable. The *stability* *region* of a network is the closure of the set of all stabilizable rates.

In order to characterize the stability region, we need to introduce the notion of feasible flows. Recall that $\Pi(\mathcal{F};\mathcal{E},v)$ is the marginal probability of success for set $\mathcal{F}$ given $\mathcal{E}$ is transmitted. In the following, to denote the marginal probability of success over link l, we will use the notation $\Pi(l;\mathcal{E},v)$ instead of $\Pi(\{l\};\mathcal{E},v)$.

Definition Rate $\lambda = (\lambda_{ij} \geq 0 : i,j \in \mathcal{N})$ is called *feasible* if there exist a scheduling vector $G = (p(\mathcal{E};v) \geq 0 : v \in \mathcal{V}, \mathcal{E} \in \mathscr{E})$ and a routing vector $H = (x_P : P \in \mathscr{P})$ such that

$$\sum_{i,j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{ij}:\, l\in P} x_P \lambda_{ij} \leq \sum_{v\in\mathcal{V},\mathcal{E}\in\mathscr{E}} \Pi(l;\mathcal{E},v)p(\mathcal{E};v)p(v) \quad (6)$$

holds for all $l \in \mathcal{L}$. The *flow region* is the closure of the set of all feasible rates.

Our motivation for defining feasibility is the following. Consider an RTD policy with G and H. When the arrival rate is λ and the routing is done according to H, the aggregate traffic on link $l \in \mathcal{L}$ is

$$\sum_{i,j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{ij}:\, l\in P} x_P \lambda_{ij}.$$

The expected number of successful transmissions on link l is

$$\sum_{v\in\mathcal{V},\mathcal{E}\in\mathscr{E}} \Pi(l;\mathcal{E},v)p(\mathcal{E};v)p(v) \quad (7)$$

given that $p(\mathcal{E};v)$ is the fraction of time slots the transmission set $\mathcal{E}$ is used when the network is in state v. The RTD policy chooses each schedule $\mathcal{E}$ with probability $p(\mathcal{E};v)$, but sometimes some other schedule $\mathcal{E}' \subset \mathcal{E}$ may be applied since nodes may run out of packets to transmit. In such cases, due to assumption (A1), the success probability over link $l \in \mathcal{E}'$ does not decrease, and (7) can be thought as a worst case success rate. Eqn. (6) ensures that the traffic load over each link is less than its worst case success rate. The following lemma asserts that this intuitive condition is sufficient for network stability.

Lemma 1: If λ is feasible with scheduling vector G and routing vector H, then the RTD policy specified by G, H, λ stabilizes the network with arrival rate $(1-\epsilon)\lambda$ for all $\epsilon > 0$.

Proof: We use the so-called dominant system approach (*e.g.,* [14], [15], [17]–[19]). That is, we first analyze a *heavy loaded* network where the nodes always have packets to send. The heavy load assumption decouples the network stability problem into a series of queues problem whose stability is established using Loynes' theory [36]. We then provide a *stochastic ordering* relation (*e.g.,* [37]) between the normal network (where the queues can be empty) and the heavy loaded network. The stochastic order says that the buffer overflows are more likely in the heavily loaded network. Therefore, the stability of the heavy loaded network implies the stability of the normal one.

The more complicated, somewhat unexpected, part in the proof is the stochastic ordering relation (Lemma 3) where we need the assumption (A1). This assumption formalizes the notion that the more is the number of transmissions, the less the probability of success; therefore, the queue lengths tend to be higher in the heavy loaded network. To prove the stochastic order, we construct a probability space (where both the normal network and the heavy loaded network lives) in which it is shown that the number of packets waiting in the heavy loaded network is more than the number waiting in the normal network with probability one.

See Appendix A for details. □

Theorem 1: The stability and the flow regions are identical.

Proof: Lemma 1 shows that the rates inside the flow region are stabilizable. For the converse, see Appendix E. □

From the definition of flow region in (6), we note that only the *marginal* link success probabilities $\Pi(l;\mathcal{E},v)$ —not the joint probabilities— determine the flow region and, therefore by Theorem 1, the stability region.

Theorem 1 suggests a way to think about stability problems. To check if rate λ is stabilizable, we need to find a distribution G over schedules, and then we need to route packets according to another distribution H such that the traffic over each link is less than its success rate, *i.e.,* rate λ is feasible with G and H. This approach is essentially similar to the standard flow approach (*e.g.,* Ford and Fulkerson [38]) that assigns a fixed capacity to each network link, and routes as much flow as possible from the sources to their destinations without violating the link capacities. In our network the link capacities are determined by the link success rates of a scheduling vector G. For fixed G, the standard flow approach [38] can be applied to obtain achievable rates. Characterization of the stability region using flows is typical in many other wired and wireless stochastic networks (*e.g.,* [2]–[9]).

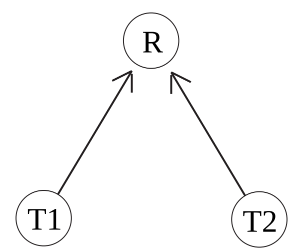

Fig. 1. An up-link network

It is interesting to notice that Theorem 1 crucially depends on assumption (A1). In general, it is true that the stability region is inside the flow region, but in networks violating (A1) feasible rates may not be stabilizable. For instance, consider the network depicted in Fig. 1. Nodes T1 and T2 want to transmit packets to node R. The network has a single state, and there is a single class of traffic intended for node R. The channel reception probabilities are such that if the transmission is $\mathcal{E} = \{(T1,R),(T2,R)\}$, then the reception is $\mathcal{F} = \{(T1,R)\}$ with probability 1. If $\mathcal{E}$ is $\phi, \{(T1,R)\}$ or $\{(T2,R)\}$, then $\mathcal{F} = \phi$ with probability 1. Namely, the packets of $T1$ are successfully received only if $T2$ transmits at the same time, but in any case $T2$'s packets are not successfully received. The flow region can be obtained as $\{(\lambda_1, 0) : 0 \leq \lambda_1 \leq 1\}$ (the i'th entry shows the rate achievable by the i'th transmitter). However, the stability region is the set $\{(0,0)\}$. To see this, observe that if the arrival rate of $T1$ is positive then the arrival rate of $T2$ should be also positive; this is so, since the stabilization of $T1$ requires $T2$ to transmit simultaneously. However, if $T2$ has positive arrival rate, it goes unstable whatsoever. Hence, in any case, one of the queues go unstable if $T1$ or $T2$ has positive arrival rate. The assumption (A1) does not hold in this network since the success probability of $T1$ increases when $T2$ transmits simultaneously. From this example we see that some additional conditions on the channel π are required in order to have the stability and the flow regions the same.

Some familiar results about slotted ALOHA, [19, Thm. 1] and, in part, [14, Thm. 4] can be obtained as special cases of Lemma 1. This is because the slotted ALOHA protocol [39, p. 348] can be viewed as a special RTD policy. In a slotted ALOHA network, in case all nodes are backlogged, every node flips a coin and chooses to transmit or to listen with a certain probability. If a node decides to transmit, the neighbor to transmit can be picked randomly according to a fixed probability distribution. Such a mechanism is a special case of assigning a probability distribution over the set of schedules—the medium access in RTD. However, note that the scheduling vector G corresponding to the slotted ALOHA has a *product* form since the transmission decisions are done independently at different nodes (the decisions are allowed to be dependent in general RTD). Further results about the stability of slotted ALOHA with probabilistic receptions can be found in [32].

C. Capacity

In this subsection, we define the achievability of packet delivery rates and the capacity region. We then argue that the capacity and the stability regions are the same in networks satisfying assumption (A1).

In the capacity context each packet is identified with its source and destination (a packet with source i and destination j will be called an (i,j)-packet). The arrivals are not random. Nonetheless, the network starts with infinite number of packets waiting delivery at the source nodes, *i.e.*, every node i has infinitely many (i,j)-packets $\forall j \neq i$. Let $W_{ij}(t)$ be the number of (i,j)-packets successfully received by node j in slot t.

Definition Rate $\lambda = (\lambda_{ij} \geq 0 : i,j \in \mathcal{N})$ is called *achievable* (*c.f.* [1]) if there exists a control policy such that the average delivery rate is greater than λ, *i.e.*,

$$\liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} W_{ij}(t) \geq \lambda_{ij} \ \text{ for all } i,j \in \mathcal{N}$$

is satisfied with probability one. The *capacity region* is the closure of the set of achievable rates.

With this definition, we can state the following result, which is the achievability version of Lemma 1.

Lemma 2: If λ is feasible with scheduling vector G and routing vector H, then the RTD policy[4] specified by G, H, λ achieves rate $(1-\epsilon)\lambda$ for all $\epsilon > 0$.

Proof: See Appendix C. □

[4] In a network applying RTD without random arrivals, we require the source nodes to regulate their traffic entering the network. This condition is needed to make the proof easier, and it is discussed in Appendix C

Theorem 2: The capacity and the flow regions are identical.

Proof: Lemma 2 shows that the flow region is inside the capacity region. The converse is outlined in Appendix I. □

Surprisingly, Theorem 2 is valid even without assumption (A1). Let us motivate this with the example in the previous subsection. We have observed that $T1$ and $T2$ should transmit together to achieve non-zero rates. However, $T1$'s packets never get through and its buffer goes unstable when it has non-zero arrival rates. In the capacity problem, $T1$ already has infinitely many packets, and stability is not an issue. Therefore, every rate $(\lambda_1, 0),\ 0 \leq \lambda_1 \leq 1$, can be achieved if $T1$ and $T2$ transmit together in λ_1 fraction of the slots. This implies that the capacity region is $\{(\lambda_1, 0) : 0 \leq \lambda_1 \leq 1\}$ which is also the flow region. As in this example, Lemma 2 holds in networks without (A1), and the capacity and the flow regions are always the same. We further discuss why this result is true in general in Appendix D.

D. An Upper Bound Using Transport Capacity

In this subsection, we introduce the notion of transport capacity, and develop an upper bound on the achievable rates using the transport capacity. This upper bound is particularly useful in large networks where the exact computation of capacity region may not be computationally feasible.

Most of the wireless networks come with a notion of distance metric telling how close two nodes are. Some commonly used metrics are the Euclidean distance and the minimum number of hops required to reach from one node to another. Let $d(i,j)$ be the distance between nodes i and j. The distance metric $d(i,j)$ is assumed to satisfy the *triangle inequality i.e.*, for all $P \in \mathscr{P}_{ij}$,

$$d(i,j) \leq \sum_{l \in P} d(l),$$

where we use the notation $d(l)$ as a shorthand for $d(t(l), r(l))$. The usual definition of metric puts additional constraints of non-negativity and symmetry of $d(i,j)$ (*e.g.*, Rudin [40]). These constraints are not needed for the results in this paper.

The next proposition gives a necessary condition for achievability.

Proposition 1: Let rate λ be in the capacity (or equivalently, in the stability or flow) region. Then,

$$\sum_{i,j \in \mathcal{N}} \lambda_{ij} d(i,j) \leq \sum_{v \in \mathcal{V}} p(v) \sum_{l \in \mathcal{L}} d(l) \Pi(l; \mathcal{E}_v, v), \tag{9}$$

where

$$\mathcal{E}_v = \arg\max_{\mathcal{E} \in \mathscr{E}} \sum_{l \in \mathcal{L}} d(l) \Pi(l; \mathcal{E}, v).$$

Proof: Let λ be feasible with a scheduling vector G and a routing vector H. Then,

$$\begin{aligned} \sum_{i,j \in \mathcal{N}} & \lambda_{ij} d(i,j) \\ & \leq \sum_{i,j \in \mathcal{N}} \sum_{P \in \mathscr{P}_{ij}} \sum_{l \in \mathcal{L}: l \in P} x_P \lambda_{ij} d(l) \qquad (10) \\ & = \sum_{l \in \mathcal{L}} d(l) \sum_{i,j \in \mathcal{N}} \sum_{P \in \mathscr{P}_{ij}: l \in P} x_P \lambda_{ij} \end{aligned}$$

$$\leq \sum_{l\in\mathcal{L}} d(l) \sum_{v\in\mathcal{V},\mathcal{E}\in\mathscr{E}} \Pi(l;\mathcal{E},v)p(\mathcal{E};v)p(v) \quad (11)$$
$$\leq \sum_{v\in\mathcal{V}} p(v) \sum_{l\in\mathcal{L}} d(l)\Pi(l;\mathcal{E}_v,v),$$

where (10) follows from the triangle inequality and (11) holds since λ is feasible. □

An interpretation of Proposition 1 is as follows. The quantity $\sum_{i,j\in\mathcal{N}} \lambda_{ij} d(i,j)$ can be viewed as the amount of work that needs to be done for carrying packets with rate λ. Similarly, $\sum_{l\in\mathcal{L}} d(l)\Pi(l;\mathcal{E},v)$ is the expected progress, or work done, by using schedule $\mathcal{E}\in\mathscr{E}$. We call the right hand side of (9)

$$\sum_{v\in\mathcal{V}} p(v) \sum_{l\in\mathcal{L}} d(l)\Pi(l;\mathcal{E}_v,v)$$

as the *transport capacity* [1], which is the expected progress averaged over v, maximized with respect to $\mathcal{E}$ for each v. Informally speaking, Proposition 1 says that the total work that can be done by the network is always less than its transport capacity.

Definition Rate $\lambda' > 0$ is called *uniformly-achievable* if $(\lambda' 1(i\neq j) : i,j\in\mathcal{N})$ is in the capacity region, where $1(\cdot)$ is the indicator function. The *network capacity* (denoted by η) is $N-1$ times the maximum of the uniformly-achievable rates. (Multiplication by $N-1$ gives the per-node throughput, *i.e.,* the sum rate delivered from a node to the other $N-1$ nodes.)

This notion of network capacity (with a different scaling) was previously used by Toumpis and Goldsmith [41]. The following theorem readily follows from Proposition 1 and the definition of η.

Theorem 3: An upper bound on the network capacity is given by

$$\eta \leq \frac{1}{\bar{L}N} \sum_{v\in\mathcal{V}} p(v) \sum_{l\in\mathcal{L}} d(l)\Pi(l;\mathcal{E}_v,v), \quad (13)$$

where $\bar{L}$ is the average distance between two arbitrarily selected nodes, *i.e.,*

$$\bar{L} = \frac{1}{N(N-1)} \sum_{i,j\in\mathcal{N}} d(i,j),$$

and

$$\mathcal{E}_v = \arg\max_{\mathcal{E}\in\mathscr{E}} \sum_{l\in\mathcal{L}} d(l)\Pi(l;\mathcal{E},v).$$

Proof: Substitute $\lambda_{ij} = \lambda'$ in (9). After rearranging (9), observe that $(N-1)\lambda'$ is less than the right hand side of (13) for all λ'. □

Theorem 3 is the main tool we will use for upper bounding the capacities of the regular networks in Part II. Recall that in our original formulation of feasibility we need to check the existence of two things: a scheduling vector and a routing vector. Theorem 3 simplifies our job by eliminating the routing vectors from the formulation. Even though the upper bound provided by Theorem 3 is not achievable in general, we will see that it is achievable in regular networks. Furthermore, we will observe another advantage of Theorem 3 in regular networks: It holds not only for the Euclidean metric but also for any metric satisfying the triangle inequality.

IV. Conclusion

In this paper, we considered a general probabilistic model for wireless networks, and studied the network stability as well as the network capacity. We have characterized the stability and capacity regions using network flows. We have also introduced a class of policies sufficient to achieve stability and capacity. In the considered model the capacity and stability regions are not identical in general. However, we have given a mild condition under which the stability and the capacity regions are the same. We have also provided a simple necessary condition for achievability using the transport capacity.

In Part II [20], we apply the RTD policies and the flow analysis to the ring and Manhattan networks. We obtain a closed-form expression for the capacity of Manhattan networks and analyze the impact of link fading, link state information and the topology information on achievable rates. We also compare a suboptimal scheme that uses ALOHA as its medium access to the optimal policy that jointly optimize medium access and routing. We finally examine the effect of variable connectivity on the capacity of Manhattan and ring networks.

Appendix

A. Lemma 1: Feasibility Implies Stability

Throughout this section we consider the stability setting: the packet arrivals are random, and the nodes start operation with empty queues. We will analyze an RTD policy determined by scheduling vector G, routing vector H and $\lambda = (\lambda_{ij} : i,j\in\mathcal{N})$. A network with an RTD policy operates as if the packets are classified according to their routes.[5] We call a packet with route P as a P-packet; let $q(P)$ denote the destination node of route P. We will use the following notation for (1):

$$n_{i,P}(t+1) = \begin{cases} n_{i,P}(t) - \sum_{l\in\mathcal{L}:\ t(l)=i} F_{l,P}(t,E(t),v(t)) \\ \quad + \sum_{l\in\mathcal{L}:\ r(l)=i} F_{l,P}(t,E(t),v(t)) + A_{i,P}(t), & \text{if } i\neq q(P) \\ 0, & \text{if } i = q(P). \end{cases} \quad (16)$$

Here, $n_{i,P}$ denotes the number of P-packets at the i'th node; $A_{i,P}, F_{l,P}, E_{l,P}$ are defined similarly. We replace the notation for $F_{l,P}(t)$ by $F_{l,P}(t,E(t),v(t))$ that also indicates the state $v(t)$ and the transmissions $E(t)$.

Let $D_{l,P}(t)$ be equal to 1 if the RTD policy has chosen to transmit a packet with route P over link l, and 0 otherwise. Recall that even though $D_{l,P}(t)$ is 1, a packet over link l is not transmitted if the scheduled transmitter does not have any P-packets *i.e.,* $n_{t(l),P} = 0$. This definition helps us to express the operation of the RTD policy concisely:

$$E(t) = (\, E_{l,P}(t) = D_{l,P}(t)1(n_{t(l),P}(t) > 0)\ : l\in\mathcal{L}, P\in\mathscr{P}). \quad (17)$$

Now, we are in a position to describe the heavy loaded network. Define $D(t) = (D_{l,P}(t) : l\in\mathcal{L}, P\in\mathscr{P})$. The queue

[5] The arrival rates scale accordingly, *i.e.,* for $\lambda = (\lambda_{ij} : i,j\in\mathcal{N})$, the arrival rate of packets with route $P\in\mathscr{P}_{ij}$ is $\lambda_{ij}x_P$.

lengths $n^*_{i,P}(t)$ in a heavy loaded network evolve as follows:

$$\begin{aligned}n^*_{i,P}(t+1) &= n^*_{i,P}(t) + A_{i,P}(t) \\ &\quad - \sum_{l\in\mathcal{L}:\ t(l)=i} F_{l,P}(t, D(t), v(t))\ 1(n^*_{t(l),P}(t) > 0) \\ &\quad + \sum_{l\in\mathcal{L}:\ r(l)=i} F_{l,P}(t, D(t), v(t))\ 1(n^*_{t(l),P}(t) > 0),\end{aligned} \quad (18)$$

if $i \neq q(P)$. To obtain (18) from (16), we moved the $1(\cdot)$ in (17) to the outside of $F(\cdot)$ as a multiplicative factor. In the normal network the nodes which are scheduled to transmit may not transmit since they may not have a packet to transmit. In the heavy loaded network, on the contrary, the receptions $F(\cdot)$ are determined as if the set of transmissions is $D(t)$ in each slot. We use the term "heavy loaded" because $E(t) = D(t)$ is possible only if every scheduled transmitter has packets waiting all the time, *i.e.*, the nodes are heavily loaded.

The following proposition asserts the stability of the heavy loaded network.

Proposition 2: Let λ be feasible with G and H. For $\epsilon > 0$, let the arrival rate of P-packets ($P \in \mathscr{P}_{ij}$) be

$$\lambda_P = (1-\epsilon)\lambda_{ij} x_P.$$

Then, the nodes in the heavy loaded network are stable, *i.e.*, for each i, P there exists $W(\cdot)$ such that

$$\lim_{t\to\infty} \Pr\{n^*_{i,P}(t) \leq \theta\} = W(\theta) \ \text{ and } \ \lim_{\theta\to\infty} W(\theta) = 1. \quad (20)$$

Furthermore, $\frac{1}{t} n^*_{i,P}(t) \to 0$ almost surely as $t\to\infty$.

Proof: In a network with an RTD policy, packets with a route P follow a series of queues. The stability of the network follows from a standard application of Loynes' theory [36] for series of queues. In the following, we will discuss the rationale behind the proposition.

Let l be a link in path P. The analysis of the heavy loaded network is much simpler than the normal network, because the event of successful transmission of P-packets over link l does not depend on queue lengths at other nodes. That is, regardless of what is happening at the other queues, the P-packets are successfully transmitted over link l according to the process $(F_{l,P}(t, D(t), v(t)) : t \in \mathcal{Z}_+)$, and it is the mean of this process that determines the stability.

From the definition of RTD policies, it follows that the mean of $F_{l,P}(\cdot)$ is

$$\mathbb{E}\{\ F_{l,P}(t, D(t), v(t))\ \} = Q(P,l) \sum_{v\in\mathcal{V},\mathcal{E}\in\mathscr{E}} \Pi(l;\mathcal{E},v) p(\mathcal{E};v) p(v).$$

If $P \in \mathscr{P}_{ij}$, then because of feasibility (6) and the definition of $Q(P,l)$,

$$\lambda_{ij} x_P \leq \mathbb{E}\{\ F_{l,P}(t, D(t), v(t))\ \}.$$

Moreover, since the arrival rate of P-packets λ_P is strictly smaller than $\lambda_{ij} x_P$, it follows that

$$\lambda_P < \mathbb{E}\{\ F_{l,P}(t, D(t), v(t))\ \}.$$

This is the condition required for stability in Loynes' theory: arrival rate for class P (left hand side) is strictly smaller than the expected number of P-packets transmitted over link l (right hand side) for each link $l \in P$. The second statement of the proposition, $\frac{1}{t} n^*_{ij}(t) \to 0$ almost surely, follows from the convergence arguments in Section 2.32 [36]. □

Next lemma gives the previously mentioned stochastic ordering relation.

Lemma 3: For each $P \in \mathscr{P}$, the total number of P-packets in the heavy loaded network is stochastically larger than the total number P-packets in the normal network, *i.e.*,

$$\Pr(\sum_{i\in\mathcal{N}} n^*_{i,P}(t) > \theta) \ \geq \ \Pr(\sum_{i\in\mathcal{N}} n_{i,P}(t) > \theta), \quad (21)$$

for all t and θ. Moreover, under the conditions in Proposition 2, $\frac{1}{t} n_{i,P}(t) \to 0$ almost surely as $t\to\infty$.

Proof: See Appendix B. □

Now we can prove the stability of the network without heavy loaded transmissions. Under the conditions in Proposition 2, (20) implies that each $n^*_{i,P}(t)$ is a substable sequence. Sums of nonnegative substable sequences is substable (see Szpankowski [17]), therefore, $\sum_{i\in\mathcal{N}} n^*_{i,P}(t)$ is substable. From (21), this implies that $\sum_{i\in\mathcal{N}} n_{i,P}(t)$ is substable. Since a nonnegative sequence smaller than a substable sequence is substable, each $n_{i,P}(t)$ in the normal network is substable. Again using the fact that sum of substable sequences is substable, we see that each $n_i(t) = \sum_{P\in\mathscr{P}} n_{i,P}(t)$ is substable. Hence, the proof of Lemma 1 is complete.

B. Stochastic Ordering - Proof of Lemma 3

When two random vectors X and Y have the same distribution, we write $X \overset{d}{=} Y$. Let $\{0,1\}^{\mathcal{LC}}$ be the set of all vectors of the form $X = (X_{l,P} \in \{0,1\} : l \in \mathcal{L}, P \in \mathscr{P})$. For deterministic X, Y in $\{0,1\}^{\mathcal{LC}}$ we say that $X \leq Y$ if $X_{l,P} \leq Y_{l,P}$ for all l, P. Define the product vector as

$$XY = (X_{l,P} Y_{l,P} : l \in \mathcal{L}, P \in \mathscr{P}).$$

Let X, Y be two $\{0,1\}^{\mathcal{LC}}$ valued random vectors such that

$$\Pr\{X \geq Z\} \leq \Pr\{Y \geq Z\} \text{ for all } Z \in \{0,1\}^{\mathcal{LC}}.$$

Then, X is said to be smaller than Y *in the usual stochastic order* (denoted by $X \leq_{\text{st}} Y$).

There are several ways to look at the stochastic ordering relations. One approach is provided by the definition above which does not restrict X and Y to be defined in the same probability space. However, there is another, sometimes more convenient, way of looking at stochastic order. If $X \leq_{\text{st}} Y$, then this equivalent approach (given as Theorem 4.B.1 in [37]) constructs new random vectors $\hat{X}$ and $\hat{Y}$ in some probability space such that $\hat{X} \overset{d}{=} X$, $\hat{Y} \overset{d}{=} Y$ and $\hat{X} \leq \hat{Y}$ with probability 1. In other words, we can view the stochastic order as the usual order in an appropriate probability space.

In the proof of Lemma 3, it is the second approach we will be using. We will construct two new stochastic processes (one mimicking the normal network, the other mimicking the heavy loaded one) such that the stochastic order relation given in

Lemma 3 can be viewed as the usual order in some appropriate space. Our plan is as follows. We will first show the existence of this new space. Then we will observe the results in Lemma 3 in the new space, and discuss the equivalence of the newly constructed network and the real network.

One useful property of the stochastic order is provided below.

Theorem 4: Let X, Y_i be $\{0,1\}^{\mathcal{LC}}$ valued random vectors, and Z_i be a (deterministic) vector in $\{0,1\}^{\mathcal{LC}}$, $i = 1, 2, \cdots, r$. If

$$XZ_i \leq_{\rm st} Y_i Z_i \quad i = 1, 2, \cdots, r$$

then there exists random vectors $\hat{X}, \hat{Y}_i$ defined on the same probability space such that $\hat{X} \stackrel{d}{=} X$, $\hat{Y}_i \stackrel{d}{=} Y_i$ and $\hat{X} Z_i \leq \hat{Y}_i Z_i$ with probability 1, $i = 1, 2, \cdots, r$.

Proof: This theorem is a straightforward extension of Theorem 4.B.1 in [37], and its proof is omitted. □

The next lemma is the major step in constructing the new network processes. Recall that for vectors E, F, D in $\{0,1\}^{\mathcal{LC}}$, the sets $\mathcal{E}, \mathcal{F}$ and $\mathcal{D}$ are defined as $\mathcal{E} = \{l \in \mathcal{L} : E_{l,P} = 1 \text{ for some } P \in \mathscr{P}\}$, $\mathcal{F} = \{l \in \mathcal{L} : F_{l,P} = 1 \text{ for some } P \in \mathscr{P}\}$, $\mathcal{D} = \{l \in \mathcal{L} : D_{l,P} = 1 \text{ for some } P \in \mathscr{P}\}$.

Lemma 4: There exists a set of $\{0,1\}^{\mathcal{LC}}$ valued random vectors

$$\mathcal{I} = \{I(E,D,v) : E, D \in \{0,1\}^{\mathcal{LC}}, E \leq D, v \in \mathcal{V}\},$$

defined on the same probability space, such that for every random vector $I(E,D,v) \in \mathcal{I}$,

i) $I(E,D,v)$ is distributed according to $\pi(\cdot\,;\mathcal{E},v)$. That is, $\Pr\{I(E,D,v) = F\} = \pi(\mathcal{F};\mathcal{E},v)$ for every vector F in $\{0,1\}^{\mathcal{LC}}$.

ii) $I(D,D,v)E \leq I(E,D,v)$ with probability 1.

Proof: Let X, Y, Z be vectors in $\{0,1\}^{\mathcal{LC}}$ such that $X \leq Y$. Observe that

$$ZY \geq X \text{ if and only if } Z \geq X. \tag{22}$$

Let $I(D,D,v), I(E,D,v)$ be random vectors satisfying i). Observe that

$$I(E,D,v)E = I(E,D,v). \tag{23}$$

This is because $\Pr\{I(E,D,v) = F\} > 0$ only if $F \leq E$, which implies that $FE = F$.

We claim that

$$I(D,D,v)E \leq_{\rm st} I(E,D,v)E. \tag{24}$$

To see this, consider a vector F in $\{0,1\}^{\mathcal{LC}}$ such that $F \leq E$.

$$\begin{aligned}
\Pr\{I(D,D,v)E \geq F\} &= \Pr\{I(D,D,v) \geq F\} \\
&= \Pi(\mathcal{F};\mathcal{D},v) \\
&\leq \Pi(\mathcal{F};\mathcal{E},v) \\
&= \Pr\{I(E,D,v) \geq F\} \\
&= \Pr\{I(E,D,v)E \geq F\}.
\end{aligned}$$

The first equality is due to (22). The second and third equalities are because $I(D,D,v), I(E,D,v)$ are random vectors satisfying i). The last equality is due to (23). The inequality is because of $E \leq D$ and assumption (A1).

We can apply Theorem 4 as a result of (24). For this, fix some D and v. Set $X = I(D,D,v)$, $Y_i = I(E,D,v)$, $Z_i = E$ such that each $E \leq D$ corresponds to a different Z_i. Theorem 4 says that there exists a probability space such that we can define random vectors $\{I(E,D,v) : E \leq D\}$ satisfying i) and $I(D,D,v)E \leq I(E,D,v)E$. Also notice that $I(E,D,v)E = I(E,D,v)$ holds due to (23).

We have shown the existence of the space of random vectors $\{I(E,D,v) : E \leq D\}$ satisfying i) and ii) for fixed D and v. These sets of random vectors for different D and v can be put into the same probability space since the involved random vectors are discrete. Therefore, a set of random vectors $\mathcal{I}$ satisfying i) and ii) exists in some probability space and the lemma follows. □

Next, we will define new stochastic processes. Suppose that we have a vector valued stochastic process $\{\mathcal{I}(t) : t \in \mathcal{Z}_+\}$ such that each entry

$$\mathcal{I}(t) = \{I(t,E,D,v) : E, D \in \{0,1\}^{\mathcal{LC}}, E \leq D, v \in \mathcal{V}\},$$

is i.i.d. distributed same as $\mathcal{I}$ in the previous lemma. We define the *normal m-network* with the following:

$$\begin{aligned}
m_{i,P}(t+1) = m_{i,P}(t) &- \sum_{l \in \mathcal{L}:\, t(l)=i} I_{l,P}(t, E(t), D(t), v(t)) \\
&+ \sum_{l \in \mathcal{L}:\, r(l)=i} I_{l,P}(t, E(t), D(t), v(t)) \; + A_{i,P}(t).
\end{aligned}$$

Above, the notation $m_{i,P}(t)$ refers to the queue length, and the rest are as before. The queue length processes in the normal network (16) and the normal m-network are indistinguishable. That is, it can be easily checked by induction over r that the joint distribution of queue lengths $\{(n_{i,P}(t) : i \in \mathcal{N}, P \in \mathscr{P}), t = 1, 2, \cdots, r\}$ and $\{(m_{i,P}(t) : i \in \mathcal{N}, P \in \mathscr{P}), t = 1, 2, \cdots, r\}$ are the same for all r. This result basically follows from property i) of the previous lemma *i.e.*, each $I(t,E,D,v)$ is distributed according to $\pi(\ \,;\mathcal{E},v)$. Since all joint queue length distributions are the same it follows that

$$\Pr(\sum_{i \in \mathcal{N}} n_{i,P}(t) > \theta) = \Pr(\sum_{i \in \mathcal{N}} m_{i,P}(t) > \theta) \tag{25}$$

for all θ. Moreover,

$$\frac{1}{t} n_{i,P}(t) \to 0 \text{ w.p.1 if and only if } \frac{1}{t} m_{i,P}(t) \to 0 \text{ w.p.1}, \tag{26}$$

as $t \to \infty$, where w.p.1 stands for with probability 1.

Similarly, we define the heavy loaded m-network as:

$$\begin{aligned}
&m^*_{i,P}(t+1) = m^*_{i,P}(t) + A_{i,P}(t) \\
&- \sum_{l \in \mathcal{L}:\, t(l)=i} I_{l,P}(t, D(t), D(t), v(t))\; 1(m^*_{t(l),P}(t) > 0) \\
&+ \sum_{l \in \mathcal{L}:\, r(l)=i} I_{l,P}(t, D(t), D(t), v(t))\; 1(m^*_{t(l),P}(t) > 0),
\end{aligned}$$

if $i \neq q(P)$. Just as the normal m-network mimics the behavior of the normal network, the heavy loaded m-network mimics the heavy loaded network (18). That is, the joint distribution of queue lengths $\{(n^*_{i,P}(t) : i \in \mathcal{N}, P \in \mathscr{P}), t = 1, 2, \cdots, r\}$ and $\{(m^*_{i,P}(t) : i \in \mathcal{N}, P \in \mathscr{P}), t = 1, 2, \cdots, r\}$ are the same for all r. Therefore, it is true that

$$\Pr(\sum_{i\in\mathcal{N}} n^*_{i,P}(t) > \theta) = \Pr(\sum_{i\in\mathcal{N}} m^*_{i,P}(t) > \theta) \tag{27}$$

for all θ. Moreover,

$$\frac{1}{t} n^*_{i,P}(t) \to 0 \text{ w.p.1 if and only if } \frac{1}{t} m^*_{i,P}(t) \to 0 \text{ w.p.1,} \tag{28}$$

as $t \to \infty$.

Next, we will see that

$$\sum_{i\in\mathcal{N}} m^*_{i,P}(t) \ \geq\ \sum_{i\in\mathcal{N}} m_{i,P}(t), \tag{29}$$

holds with probability 1, for all P, t. This inequality is due to the property ii) of the last lemma: if $E_{l,P}(t) \cdot I_{l,P}(t, D(t), D(t), v(t)) = 1$ for some i, P then $I_{l,P}(t, E(t), D(t), v(t)) = 1$. Observe that $E_{l,P}(t) \cdot I_{l,P}(t, D(t), D(t), v(t)) = 1$ if and only if $E_{l,P}(t) = 1$ (a P-packet is transmitted over link l in the normal m-network) and $I_{l,P}(t, D(t), D(t), v(t)) = 1$ (a P-packet is successfully transmitted over link l in the heavy loaded m-network if $m^*_{t(l),P}(t) > 0$). According to Property ii) these two events imply that $I_{l,P}(t, E(t), D(t), v(t)) = 1$ (P-packet is successfully transmitted over link l in the normal m-network). That is, we can simply say that if a packet is successfully transmitted in the heavy loaded m-network in slot t, then in the normal m-network either a packet is successfully transmitted in slot t, or the transmitter queue is empty, which means that all packets have already been transmitted. This reasoning, by induction over t, leads to (29).

Because of (29),

$$\Pr(\sum_{i\in\mathcal{N}} m^*_{i,P}(t) > \theta) \ \geq\ \Pr(\sum_{i\in\mathcal{N}} m_{i,P}(t) > \theta) \tag{30}$$

holds for all θ. Equations (30), (25) and (27) give (21).

In Lemma 2 we have shown that $\frac{1}{t} n^*_{i,P}(t) \to 0$ with probability 1 as $t \to 0$. Due to (28), we have $\frac{1}{t} m^*_{i,P}(t) \to 0$ with probability 1. Since the queue lengths are nonnegative processes, inequality (29) gives $\frac{1}{t} m_{i,P}(t) \to 0$ with probability 1. And, as a result of (26), it is true that $\frac{1}{t} n_{i,P}(t) \to 0$ with probability 1, as required.

C. Proof of Lemma 2

First, let's assume that the arrivals in the network are random as considered in Appendix A. Lemma 3 shows that $\frac{1}{t} n_{i,P}(t) \to 0$, for all $i \in \mathcal{N}, P \in \mathscr{P}$. This implies that the delivery rate of P-packets is equal to the arrival rate $\lambda_P = (1-\epsilon)\lambda_{ij} x_P$. Therefore, the total delivery rate of packets from i to j is $\sum_{P\in\mathscr{P}_{ij}} \lambda_P = (1-\epsilon)\lambda_{ij}$.

In the capacity problem, every node has infinitely many packets waiting to be delivered, and the arrivals are not random. However, if the source nodes *regulate* the traffic incoming to the network and operate as if the arrivals are random, then the result in the previous paragraph is applicable. What we mean by regulation is that node i should introduce its (i, j)-packets into the network with rate λ_{ij} according to a stationary and ergodic process. Given that the nodes operate in this way the rate $(1-\epsilon)\lambda$ is achieved by the RTD policy, and Lemma 2 follows.

D. Achievability of Flow Region Without Assumption (A1)

For the argument in the previous section we need assumption (A1), which is used in Lemma 3. However, it is in general true that all rates inside the flow region are achievable. To see this, we will consider a slightly modified form of RTD policies: if a P-packet chosen for transmission over link l (*i.e.,* $D_{l,P}(t) = 1$), but the transmitter node $t(l)$ doesn't have any P-packet, then let node $t(l)$ transmit another packet with source $t(l)$ and destination $r(l)$ over link l (there are infinitely many such packets in node $t(l)$'s buffer). Besides these extra packet transmissions, the network under this policy operates no different from a heavy loaded network; specifically, the reception statistics is determined according to the set of transmissions $\mathcal{D}(t) = \{l \in \mathcal{L} : D_{l,P}(t) = 1 \text{ for some } P\}$, which is the same in the heavy loaded network. The results of Proposition 2 hold also for this network. Proposition 2 implies that delivery rate λ_P is achieved for each $P \in \mathscr{P}$, and therefore $(1-\epsilon)\lambda$ is achieved.

E. Theorem 1: Stability Implies Feasibility

In this appendix we prove the converse part of Theorem 1. Our proof is constructive. We will consider a stable network with arrival rate λ, and by using certain statistics of the network, we will construct a scheduling vector G and a routing vector H that make $\lambda - \epsilon 1_\lambda \geq 0$ feasible, where $\epsilon > 0$, and 1_λ is a shorthand for $(\,1(\lambda_{ij} > 0) \ : \ i, j \in \mathcal{N})$.

By adding up equations for $t = 1, 2, \cdots, T$ in (1), we see that

$$n_{ij}(T) = \sum_{t=0}^{T-1} \left[A_{ij}(t) + \sum_{l\in\mathcal{L}:\ r(l)=i} F_{lj}(t) - \sum_{l\in\mathcal{L}:\ t(l)=i} F_{lj}(t) \right] \tag{31}$$

holds for all $T \in \mathcal{Z}_+$, $i \neq j$. The following lemma relates stability with the expected queue length.

Lemma 5: If the network is stable, then for all $i \in \mathcal{N}$,

$$\frac{1}{t}\mathbb{E} n_i(t) \to 0 \ \text{ as } \ t \to \infty.$$

Proof: See Appendix F □

Suppose that at time instant $T \in \mathcal{Z}_+$

$$\mathbb{E}\{\frac{1}{T} \sum_{i\in\mathcal{N}} n_i(T)\} < \epsilon \tag{32}$$

is satisfied. Existence of such a T is guaranteed by the previous Lemma. Define

$$p(\mathcal{E}; v) \triangleq \frac{1}{T} \sum_{t=0}^{T-1} \Pr\{\mathcal{E}(t) = \mathcal{E} | v(t) = v\}, \tag{33}$$

and the scheduling vector $G = (p(\mathcal{E}; v) : v \in \mathcal{V}, \mathcal{E} \in \mathscr{E})$.

Lemma 6: The scheduling vector G defined by (33) satisfies

$$\sum_{j\in\mathcal{N}} \mathbb{E}\{\frac{1}{T} \sum_{t=0}^{T-1} F_{lj}(t)\} = \sum_{v\in\mathcal{V}, \mathcal{E}\in\mathscr{E}} \Pi(l; \mathcal{E}, v) p(\mathcal{E}; v) p(v),$$

for all $l \in \mathcal{L}$.

Proof: See Appendix G □

Define

$$e_{lj} = \mathbb{E}\{\frac{1}{T}\sum_{t=0}^{T-1} F_{lj}(t)\},$$
$$\gamma_{ij} = \mathbb{E}\{\frac{1}{T} n_{ij}(T)\}.$$

The next proposition provides a routing vector H which will be used to show the feasibility of $\lambda - \epsilon 1_\lambda$.

Proposition 3: Given $\epsilon > 0$ and the vectors $(e_{lj} \geq 0 : l \in \mathcal{L}, j \in \mathcal{N})$, $\lambda = (\lambda_{ij} \geq 0 : i, j \in \mathcal{N})$, $\gamma = (\gamma_{ij} \geq 0 : i, j \in \mathcal{N})$ such that for all $i, j \in \mathcal{N},\ i \neq j$,

$$\text{(i)}\quad \lambda_{ij} - \gamma_{ij} = \sum_{l\in\mathcal{L}:\ t(l)=i} e_{lj} - \sum_{l\in\mathcal{L}:\ r(l)=i} e_{lj}, \tag{34}$$

$$\text{(ii)}\quad \lambda_{jj} = \gamma_{jj} = \sum_{l\in\mathcal{L}:\ t(l)=j} e_{lj} = 0, \tag{35}$$

$$\text{(iii)}\quad \sum_{i,j\in\mathcal{N}} \gamma_{ij} < \epsilon, \tag{36}$$

$$\text{(iv)}\quad \lambda - \epsilon 1_\lambda \geq 0, \tag{37}$$

are satisfied. Then, there exists a routing vector $H = (x_P \geq 0 : P \in \mathscr{P})$ such that

$$\sum_{i,j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{ij}:\ l\in P} x_p(\lambda_{ij} - \epsilon 1(\lambda_{ij} > 0)) \leq \sum_{j\in\mathcal{N}} e_{lj} \tag{38}$$

holds for all $l \in \mathcal{L}$.

Proof: See Appendix H. □

Next, we will argue that $\lambda - \epsilon 1_\lambda$ is feasible. When we multiply both sides in (31) by $\frac{1}{T}$ and take the expectation, we see that condition 3(i) is satisfied. Condition 3(ii) holds as a result of our particular choices for γ and $(e_{lj} : l \in \mathcal{L}, j \in \mathcal{N})$. Condition 3(iii) holds because of (32). Therefore, all conditions of Proposition 3 are satisfied, and we can apply it. Proposition 3 guarantees the existence of H satisfying (38). Lemma 6 gives

$$\sum_{j\in\mathcal{N}} e_{lj} = \sum_{v\in\mathcal{V},\mathcal{E}\in\mathscr{E}} \Pi(l;\mathcal{E},v)p(\mathcal{E};v)p(v).$$

The previous equality, together with (38), implies

$$\sum_{i,j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{ij}:\ l\in P} x_P(\lambda_{ij} - \epsilon 1(\lambda_{ij} > 0)) \leq \sum_{v\in\mathcal{V},E\in\mathscr{E}} \Pi(l;\mathcal{E},v)p(\mathcal{E};v)p(v),$$

for all $l \in \mathcal{L}$. That is, $\lambda - \epsilon 1_\lambda$ is feasible.

F. *Proof of Lemma 5*

Define $G_{ij}(t) = \sum_{l\in\mathcal{L}:\ r(l)=i} F_{lj}(t)$ and $H_{ij}(t) = \sum_{l\in\mathcal{L}:\ t(l)=i} F_{lj}(t)$. With these definitions, we can write (31) as

$$n_{ij}(T) = \sum_{t=0}^{T-1} \left[A_{ij}(t) + G_{ij}(t) - H_{ij}(t)\right]. \tag{39}$$

If the network is stable, the following is satisfied.

$$\lim_{\theta\to\infty} \limsup_{t\to\infty} \Pr\{n_i(t) > \theta\} = 0, \text{ for all } i \in \mathcal{N}.$$

Now, pick an $i \in \mathcal{N}$. For all $\theta, \epsilon > 0$, there exists t_0 such that for all $t > t_0$, $\Pr\{n_i(t) > t\epsilon\} \leq \Pr\{n_i(t) > \theta\}$. First, consider the limit $t \to \infty$,

$$\limsup_{t\to\infty} \Pr\{n_i(t) > t\epsilon\} \leq \limsup_{t\to\infty} \Pr\{n_i(t) > \theta\}.$$

Then, let $\theta\to\infty$,

$$\limsup_{t\to\infty} \Pr\{n_i(t) > t\epsilon\} \leq \lim_{\theta\to\infty} \limsup_{t\to\infty} \Pr\{n_i(t) > \theta\} = 0.$$

This means that $\frac{1}{t} n_i(t) \xrightarrow{\mathcal{P}} 0$, where $\xrightarrow{\mathcal{P}}$ denotes convergence in probability. Pick an arbitrary $j \in \mathcal{N}$. Since $n_{ij}(t)$ is nonnegative and less than $n_i(t)$, $\frac{1}{t} n_{ij}(t) \xrightarrow{\mathcal{P}} 0$. Write (39) as

$$\frac{1}{t}\sum_{r=0}^{t-1} A_{ij}(r) - \frac{1}{t}\sum_{r=0}^{t-1}[H_{ij}(r) - G_{ij}(r)] = \frac{1}{t} n_{ij}(t) \xrightarrow{\mathcal{P}} 0. \tag{41}$$

Since $(A_{ij}(t) : t \in \mathcal{Z}_+)$ is ergodic,

$$\lim_{t\to\infty} \frac{1}{t}\sum_{r=0}^{t-1} A_{ij}(r) = \lambda_{ij}$$

almost surely. Hence, we have

$$J_{ij}(t) \triangleq \frac{1}{t}\sum_{r=0}^{t-1}\left[H_{ij}(r) - G_{ij}(r)\right] \xrightarrow{\mathcal{P}} \lambda_{ij}. \tag{42}$$

Observe that $|H_{ij}(t)| \leq |\mathcal{L}|$ and $|G_{ij}(t)| \leq |\mathcal{L}|$ for all t. Therefore, $|J_{ij}(t)| \leq 2|\mathcal{L}|$. This boundedness property, together with (42), implies that $\mathbb{E}J_{ij}(t) \to \lambda_{ij}$. When we take expectation of both sides in (41),

$$\frac{1}{t}\mathbb{E}n_{ij}(t) = \lambda_{ij} - \mathbb{E}J_{ij}(t) \to 0.$$

The lemma follows.

G. *Proof of Lemma 6*

The key step in the proof is observing that

$$\mathbb{E}\{\sum_{j\in\mathcal{N}} F_{lj}(t) \mid \mathcal{E}(t) = \mathcal{E}, v(t) = v\} = \Pi(l;\mathcal{E},v)$$

holds for all $\mathcal{E}, v, l, t$ (this directly follows from the definition of $\Pi(\cdot)$). The following sequence of equalities lead to the required lemma. For every $l \in \mathcal{L}$,

$$\sum_{j\in\mathcal{N}} \mathbb{E}\{\frac{1}{T}\sum_{t=0}^{T-1} F_{lj}(t)\} = \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}\{\sum_{j\in\mathcal{N}} F_{lj}(t)\}$$

$$= \frac{1}{T}\sum_{t=0}^{T-1}\sum_{v\in\mathcal{V},\mathcal{E}\in\mathscr{E}}\mathbb{E}\{\sum_{j\in\mathcal{N}}F_{lj}(t)\mid\mathcal{E}(t)=\mathcal{E},v(t)=v\}\Pr\{\mathcal{E}(t)=\mathcal{E},v(t)=v\}$$

$$= \frac{1}{T}\sum_{t=0}^{T-1}\sum_{v\in\mathcal{V},\mathcal{E}\in\mathscr{E}}\Pi(l;\mathcal{E},v)\Pr\{\mathcal{E}(t)=\mathcal{E}|v(t)=v\}p(v)$$

$$= \sum_{v\in\mathcal{V},\mathcal{E}\in\mathscr{E}}\Pi(l;\mathcal{E},v)\frac{1}{T}\sum_{t=0}^{T-1}\Pr\{\mathcal{E}(t)=\mathcal{E}|v(t)=v\}p(v)$$

$$= \sum_{v\in\mathcal{V},\mathcal{E}\in\mathscr{E}}\Pi(l;\mathcal{E},v)p(\mathcal{E};v)p(v).$$

H. Flow Vectors - Proof of Proposition 3

First, we will introduce some definitions and lemmas. The proof of Proposition 3 will be given at the end of this section. For a given network with nodes $\mathcal{N}$ and links $\mathcal{L}$, a vector $E=(e_l\geq 0: l\in\mathcal{L})$ will be called a *flow vector*. For a given flow vector E, for every $i\in\mathcal{N}$ define

$$f_i(E)\triangleq\sum_{l\in\mathcal{L}:\ t(l)=i}e_l-\sum_{l\in\mathcal{L}:\ r(l)=i}e_l$$

as the *flow* from node i into the network.

A *loop* in a network is defined as an ordered m-tuple of links $(l_1,l_2,\cdots,l_m)\in\mathcal{L}^m$ such that the following is satisfied.

(i) $r(l_m)=t(l_1)$
(ii) $r(l_i)=t(l_{i+1}),\ i=1,2,\cdots,m-1$
(iii) $(r(l_1),r(l_2),\cdots,r(l_m))$ is a path.

Denote the set of all loops with $\mathscr{L}$. In a flow vector E, the flow across a loop $L\in\mathscr{L}$ is defined as

$$flow(L,E)\triangleq\min\{e_{l_1},e_{l_2},\cdots,e_{l_m}\}.$$

A flow vector *without loops* is a flow vector E such that for any loop L, $flow(L,E)=0$.

Lemma 7: Let $E=(e_l: l\in\mathcal{L})$ be a flow vector. There exists a flow vector without loops $\hat{E}=(\hat{e}_l: l\in\mathcal{L})$ satisfying the following

(i) $0\leq\hat{e}_l\leq e_l$, for all $l\in\mathcal{L}$.
(ii) The flow of each node in E and $\hat{E}$ are the same, *i.e.,* $f_i(E)=f_i(\hat{E}),\ \forall i\in\mathcal{N}$.

Proof: For a loop $L=(l_1,l_2,\cdots,l_m)$ and a link l, we say that $l\in L$ if $l=l_k$ for some k between 1 and m. In order to prove the lemma, we will give an algorithm that eliminates all loops in E step by step.

Suppose that $\mathscr{L}=\{L_1,L_2,\cdots,L_{|\mathscr{L}|}\}$. We will use k as an index variable. Initialize $k=1$, and set $\hat{E}=E$. Then apply the following operation on the entries of $\hat{E}$,

For all $l\in L_k$, change $\hat{e}_l$ to $\hat{e}_l-flow(L_k,\hat{E})$.

We can see that after this operation (i) and (ii) are satisfied. Next, increment k and continue this procedure for $k=1,2,\cdots,|\mathscr{L}|$ one by one. In the end, we not only end up with a vector $\hat{E}$ without loops, but also the final $\hat{E}$ satisfies (i) and (ii) since after each step they are satisfied. □

Fix a $j\in\mathcal{N}$. Define

$$E_j=(e_{lj}: l\in\mathcal{L}). \tag{43}$$

In the flow vector E_j, if $\lambda_{ij}>0$ holds for some node i, then $f_i(E_j)=\lambda_{ij}-\gamma_{ij}$ as given in (34). Furthermore, due to (36) and (37) if $\lambda_{ij}>0$ then $f_i(E_j)>0$. On the other hand, if $\lambda_{ij}=0$ then $f_i(E_j)=-\gamma_{ij}\leq 0$. Hence, $\lambda_{ij}>0$ if and only if $f_i(E_j)>0$.

If $f_i(E_j)>0$, node i will be called a *source node*. If $f_i(E_j)<0$, node i will be called an *accumulation node*. If $V=(l_1,l_2,\cdots,l_n)\in\mathcal{L}^n$ and $l_{n+1}\in\mathcal{L}$ then define $V\oplus(l_{n+1})=(l_1,l_2,\cdots,l_n,l_{n+1})$, $(l_{n+1})\oplus V=(l_{n+1},l_1,l_2,\cdots,l_n\}$. If $V=\phi$, then $V\oplus(l_{n+1})=(l_{n+1})\oplus V=(l_{n+1})$.

Lemma 8: Let E_j be the flow vector defined in (43). There exists a flow vector without loops $\hat{E}_j=(\hat{e}_{lj}: l\in\mathcal{L})$ satisfying

(i) $0\leq\hat{e}_{lj}\leq e_{lj}$, for all $l\in\mathcal{L}$.
(ii) Except j, there does not exist any accumulation node in $\hat{E}_j$, *i.e.*, for all $i\neq j$, $f_i(\hat{E}_j)\geq 0$.
(iii) If $i\in\mathcal{N}$ is a source node, then $f_i(\hat{E}_j)>\lambda_{ij}-\epsilon$.

Proof: We will give an algorithm for obtaining $\hat{E}_j$ from E_j. First apply the algorithm in Lemma 7 and obtain a flow vector without loops $\hat{E}_j$ from E_j.

(A) Check if there exist an accumulation node $i\neq j$ in $\hat{E}_j$. If there is no other accumulation node, terminate.
(B) Set $V=\phi$.
(C) For node i, $f_i(\hat{E}_j)\leq 0$ is satisfied, and there exists a link l such that $r(l)=i$ and $e_l\neq 0$. Set V to $(l)\oplus V$.
  a) If $t(l)$ is a source node then for all $l\in V$ change $\hat{e}_{lj}$ to $\hat{e}_{lj}-\min\{\hat{e}_{lj}: l\in V\}$. Go to step (A).
  b) If $t(l)$ is not a source node then $f_{t(l)}(\hat{E}_j)\leq 0$. Set $i=t(l)$ and go to step (C).

In part (C), V does not form a loop at any time (*i.e.,* $V\notin\mathscr{L}$) since the flow vector does not contain any loops. Part (C) terminates in finite number of steps since there exists finitely many nodes that can be visited, and a node can not be visited more than once. The algorithm terminates in finite number of steps since there are finitely many nodes and paths in the network, and due to decrease of $\hat{e}_{lj}$'s in part (C)-(a), if a path is followed once, it can not be followed once more. We can check the properties (i) to (iii):

(i) Holds because at each step $\hat{e}_{lj}$ non-increases.
(ii) Holds since the algorithm eliminates all accumulation nodes except j.
(iii) Holds because $\sum_i\gamma_{ij}<\epsilon$, and the removal of γ_{ij} from the accumulation nodes decreases the flow from each source node at most $\sum_i\gamma_{ij}$.

□

Proof: *(Proposition 3)* For each $j\in\mathcal{N}$ define E_j as in (43), and apply the algorithm in Lemma 8 to obtain $\hat{E}_j$. Initialize $(y_P=0: P\in\mathscr{P})$. For each $i,j\in\mathcal{N}$, $\lambda_{ij}-\epsilon 1(\lambda_{ij}>0)>0$, apply the following algorithm:

(A) If $f_i(\hat{E}_j)>0$ continue, otherwise terminate.
(B) Set $V=\phi$, and $k=i$.
(C) $f_k(\hat{E}_j)\geq 0$ and there exists a link $l\in\mathcal{L}$ such that $t(l)=k$ and $\hat{e}_{lj}>0$. Set V to $V\oplus(l)$.

a) If $r(l) = j$, then set $y_V = \min\{\hat{e}_{lj} : l \in V\}$. For all $l \in V$ change $\hat{e}_{lj}$ to $\hat{e}_{lj} - \min\{\hat{e}_{lj} : l \in V\}$. Go to step (A).
b) If $r(l) \neq j$, then set $k = r(l)$ and go to step (C)

Due to Lemma 8(ii), we can make sure that in part (C) there exists a link $l \in \mathcal{L}$ such that $t(l) = k$ and $\hat{e}_{lj} > 0$.

In the end, $(y_P : P \in \mathscr{P})$ generated by the algorithm satisfies the following:

(i) Due to Lemma 8(i), $\forall j \in \mathcal{N}$,

$$\sum_{i \in \mathcal{N}} \sum_{P \in \mathscr{P}_{ij}:\, l \in P} y_P \leq e_{lj}. \tag{44}$$

(ii) Due to Lemma 8(iii), $\forall i, j \in \mathcal{N}$,

$$\lambda_{ij} - \epsilon 1(\lambda_{ij} > 0) \leq \sum_{P \in \mathscr{P}_{ij}} y_P. \tag{45}$$

For each $i, j \in \mathcal{N}$, $P \in \mathscr{P}_{ij}$, define

$$x_P \triangleq \begin{cases} \frac{y_P}{\sum_{P \in \mathscr{P}_{ij}} y_P}, & \text{if } \lambda_{ij} - \epsilon 1(\lambda_{ij} > 0) > 0 \\ 0, & \text{otherwise.} \end{cases} \tag{46}$$

We can check that for each $j \in \mathcal{N}, l \in \mathcal{L}$,

$$\begin{aligned} \sum_{i \in \mathcal{N}} \sum_{P \in \mathscr{P}_{ij}:\, l \in P} x_P(\lambda_{ij} - \epsilon 1(\lambda_{ij} > 0)) &\leq \sum_{i \in \mathcal{N}} \sum_{P \in \mathscr{P}_{ij}:\, l \in P} y_P \\ &\leq e_{lj}. \end{aligned} \tag{47}$$

The first inequality follows from (45) and (46). The second one is due to (44). As a result of (47), $(x_P : P \in \mathscr{P})$ satisfies (38), and the proposition follows. □

I. Theorem 2: Achievability Implies Feasibility

With minor modifications, we can use the same techniques employed in proving the stability implies feasibility (Appendix E). If $\lambda = (\lambda_{ij} : i, j \in \mathcal{N})$ is achievable, then by Fatou's lemma, for all $i, j \in \mathcal{N}$,

$$\begin{aligned} \lambda_{ij} &\leq \mathbb{E}\left\{ \liminf_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} W_{ij}(t) \right\} \\ &\leq \liminf_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{W_{ij}(t)\}. \end{aligned}$$

Suppose that at time $T \in \mathcal{Z}_+$, for all $i, j \in \mathcal{N}$,

$$\lambda_{ij} - \epsilon 1(\lambda_{ij} > 0) \leq \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{W_{ij}(t)\}.$$

To show that $\lambda - \epsilon 1_\lambda \geq 0$ is feasible for all $\epsilon > 0$, we can define the scheduling vector $G = (p(\mathcal{E}; v) : v \in \mathcal{V}, \mathcal{E} \in \mathscr{E})$ as in (33), and the other quantities e_{lk}, γ_{ik} similar to the ones defined in Appendix E. Then, an algorithm almost identical to the one used in Appendix H can be used to construct H showing that $\lambda - \epsilon 1_\lambda$ is feasible.

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