Stability and Capacity of Wireless Networks with Probabilistic Receptions: Part II—Regular Networks

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***Abstract*—We study the stability and capacity problems in *regular* wireless networks. In Part I, we characterized the stability and the capacity regions, and introduced policies sufficient to achieve every rate in these regions. In this part, we obtain *closed form* expressions for the capacity of Manhattan (two-dimensional grid) and ring networks (circular array of nodes). We also find the optimal (*i.e.,* capacity achieving) medium access and routing policies. Our objective in analyzing regular networks is to provide insights and design guidelines for general networks. The knowledge of the exact capacity enables us to quantify the performance loss due to suboptimal protocols (such as slotted ALOHA and random walk), and the effects of link fading and increasing connectivity.**

***Index Terms*— Wireless networks, regular networks, Manhattan, multipacket reception, capacity, stability, transport capacity, slotted ALOHA, scheduling, optimal connectivity.**

I. Introduction

In Part I of this two part paper, we characterized the capacity and stability regions of wireless networks. We also considered the *network capacity* (*i.e.*, the maximum *uniformly* achievable rate), and found that computing the network capacity requires an optimization with respect to medium access and routing policies. For general networks, however, this task is typically prohibitive due to the excessive dimensionality of the problem. As a result, one has to contend with certain asymptotics and order computations (*e.g.,* [1], [2]).

In the current paper, Part II, we aim to provide an exception to this rule. Namely, we analyze certain regular networks for which we can compute the capacity explicitly. We obtain analytical expressions for the capacity, and find the *leading coefficient* besides the scaling law. The knowledge of the coefficient enables us to make comparisons between various design choices which affect the coefficient but not the scaling law. For example, we quantify the loss incurred by suboptimal protocols and the effects of increasing connectivity. We also provide the optimal (*i.e.*, capacity achieving) medium access and routing policies for the regular networks.

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This work was supported in part by the Multidisciplinary University Research Initiative (MURI) under the Office of Naval Research Contract N00014-00-1-0564 and the ARL CTA on Communications and Networks under Grant DAAD19-01-2-011.

The Manhattan network is a two-dimensional grid with size $\sqrt{N} \times \sqrt{N}$ (Fig. 1). Every node has four neighbors, and the nodes on the edge are connected to the nodes on the opposite edge forming a torus. We use the multipacket reception (MPR) channel model [3], [4], which is previously considered as an abstraction for CDMA and multiple antenna up-link. We first show that the capacity of Manhattan network is

$$\eta = \frac{K_1}{\sqrt{N}} + O(\frac{1}{N}),$$

where the coefficient K_1 (given in Section III) depends only on the channel reception capability.[1] In case nodes can simultaneously receive multiple packets, K_1 increases but the form of η does not change.

Even though the capacity can be achieved by optimal medium access and routing, it is important to quantify the loss because of suboptimal, yet more practical, control policies. We analyze two extremes: a simple medium access method (slotted ALOHA), and a simple routing (random walking packets). We show that the maximum achievable rate with slotted ALOHA medium access and optimal routing is

$$\eta_{ALOHA} = \frac{K_2}{\sqrt{N}} + O(\frac{1}{N}),$$

where the coefficient K_2 (given in Section III-A) is smaller than K_1. On the other hand, the achievable rate with optimal medium access and random walking packets is $O(\frac{1}{N \log N})$. These results suggest that the medium access method in general does not change the order of the capacity, but the routing does change the order, and a poor routing protocol can significantly degrade the performance of large networks.

We next consider the case where the links in Manhattan network are subject to time-varying fading. We use the collision channel model with a simple model for fading; links become ON/OFF randomly in each slot (ON with probability p, OFF with probability $1-p$). A realization of this network is depicted in Fig. 1.b, where the OFF links are shown with dashed lines. In case the control policy doesn't know the states of the links before making transmission decisions, we say that the policy is without link state information (LSI). The capacity *without* LSI is shown to be

$$\eta = \frac{K_3}{\sqrt{N}} + O(\frac{1}{N}),$$

[1] When f and g are functions of N, we say that $f(N) = O(g(N))$ if there exists a scalar C such that $|f(N)| < Cg(N)$ for all N.

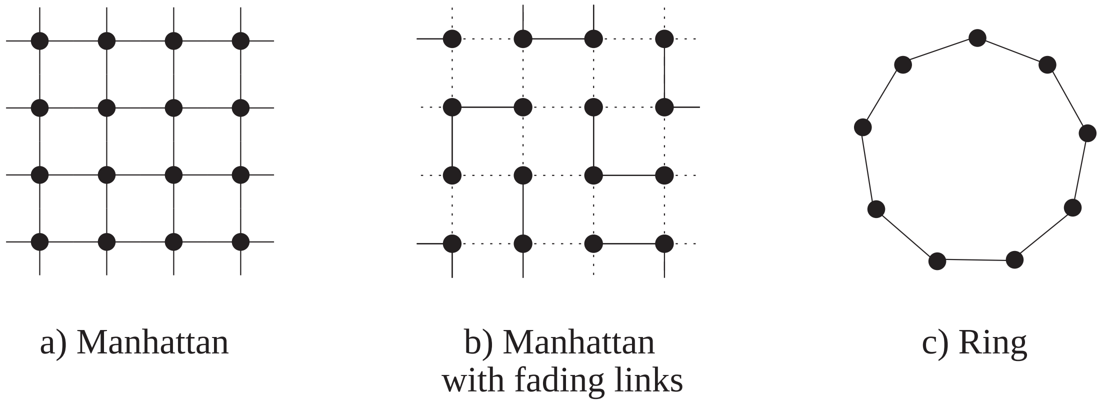

Fig. 1. Examples of regular networks

where the coefficient K_3 is given in Section IV. We develop bounds on the capacity *with* LSI (denoted by $\eta^{\#}$). Namely, we show that the ratio $\eta^{\#}/\eta$ satisfies

$$1 \leq \frac{\eta^{\#}}{\eta} \leq 2.86 + O(1/\sqrt{N}).$$

Furthermore, the bounds are reasonably tight, *i.e.,* $\eta^{\#}/\eta$ is equal to 2.5 in the limit as $p\to 0,\ N\to\infty$. Similarly, $\eta^{\#}/\eta$ is equal to 1 in the limit as $p\to 1,\ N\to\infty$. These bounds quantify the gain due to the knowledge of link state information.

Finally, we look at the optimal network-connectivity problem. Gupta and Kumar [1], and Gallager and Bertsekas [5, p. 350] discussed the trade-off between throughput vs. connectivity and argued that minimizing transmission radius while keeping the network connected leads to higher throughput. Our analysis points out two cases where choosing minimal connectivity is not optimal. In minimally connected Manhattan networks, every node has four neighbors. Let us call a Manhattan network *2-hop connected* if nodes increase their transmission radius and gets connected also with neighbors of neighbors. We show that increasing connectivity from minimal to 2-hop yields about 54% capacity increase if the nodes are capable of receiving eight packets simultaneously. Simultaneous receptions are particularly relevant for networks with spread spectrum and/or multiple antennas; in such systems, we expect performance gains from non-minimal connectivity. On the other hand, in ring networks (Fig. 1.c) the capacity is doubled by relaxing the minimal connectivity assumption. The optimal connectivity in ring goes to infinity as the network size grows. This result is true even without multipacket receptions. These examples show that minimal connectivity is not always optimal, and there are potential benefits of adaptive connectivity depending on the topology and channel usage.

The network capacity problems have been studied in several contexts. The early works focused on the computation of achievable rates with distributed protocols such as ALOHA (*e.g.,* [6], [7], [5, p. 346]) and TDMA (*e.g.,* [8], [9]). Silvester and Kleinrock analyzed the capacities of regular networks with the slotted ALOHA protocol in [6], [7]. Using the collision channel they obtained the throughput of slotted ALOHA in regular networks. They argued that the minimal connectivity is optimal in Manhattan networks with slotted ALOHA, but in ring it is not. Later, Tsybakov and Bakirov studied the stability of multi-hop ALOHA networks [10]. Besides verifying some of the results in [6] from the stability point of view, Tsybakov and Bakirov obtained other general stability conditions for arbitrary networks. Our analysis of regular networks extend Silvester and Kleinrock's results in several directions considering centralized control as well as slotted ALOHA.

Gupta and Kumar [1] initiated a formal capacity analysis of random and arbitrary networks. Unlike most of the prior studies which started with a graph model having transmission powers fixed, Gupta and Kumar considered a joint optimization of transmission powers and schedules. They showed the fundamental result that the maximum per-node throughput scales proportional to $1/\sqrt{N}$. Our setup is different from that of [1] in that the network is *ergodic*; specifically, the topology and channel qualities form an ergodic process (this is elaborated in Part I). In the ergodic network the node connections change in time according to certain statistics, and the links are not permanent as in [1]. For the regular networks in this paper we provide the capacity coefficients besides the scaling law. These coefficients, which are not apparent in [1], reveal considerable insights about the design of MAC and optimal node connectivity.

More recently, a number of other works [2], [11]–[23] studied the capacity of wireless networks from various viewpoints. Toumpis and Goldsmith [11], [12] modeled the communication channel using deterministic rate matrices, and defined the notion of capacity region. They also analyzed the capacity regions of networks considering adaptive modulation and rates depending on the channel and interference conditions. Different from Toumpis and Goldsmith, we also consider randomness in receptions. We however do not consider rate adaptation.

The organization of the paper is as follows. In the next section the multipacket reception model is introduced, and a result required from Part I is mentioned. In Section III, we compute the capacity of Manhattan networks and provide capacity achieving MAC and routing policies. The maximum achievable rate with slotted ALOHA is also computed. In Section IV, capacity with fading is analyzed. In Section V, optimal connectivity in Manhattan and ring networks is investigated. We conclude in Section VI.

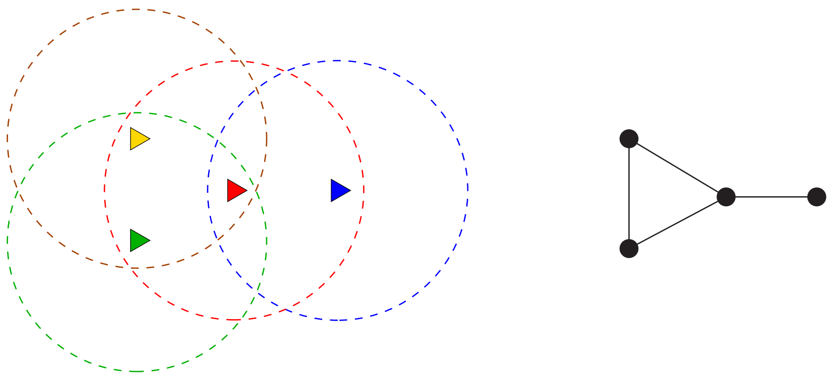

Fig. 2. Nodes and transmission ranges in a planar network are shown on the left. The right figure shows the corresponding graph.

II. Network Model

A. Multipacket Reception Channel

We consider networks represented with an undirected graph such that two nodes i and j can communicate directly only if they are connected with an edge. The graph models are traditionally used with the collision channel assumption [24]: Two nodes can communicate directly if they are within a distance r; transmission from node i to node j is successful only if there is no other transmitter within distance r to node j (see Fig. 2).

In wireless networks with CDMA (code-division multiple access) and/or multiple antennas, the collision channel assumptions do not hold. That is, the nodes might be capable of receiving multiple packets simultaneously, and there may be unexpected reception errors due to channel time-variation. To be able to consider such networks, we use the graph models with MPR [3], [4], [25]–[30]. Suppose that nodes can not transmit and receive at the same time. Each node can transmit at most one packet at a time. In each slot, a node can correctly receive a fraction of the number of transmissions in its neighborhood. The reception probabilities are given by the *Receiver MPR Matrix* $\mathbf{C}$. The entries of the MPR matrix are given as

$$C_{n,k} = \Pr\{k \text{ packets are received} \mid n \text{ packets are transmitted in the neighborhood}\}.$$

The *Receiver MPR Matrix* $\mathbf{C}$ is defined by

$$\mathbf{C} = \begin{pmatrix} C_{1,0} & C_{1,1} & & \\ C_{2,0} & C_{2,1} & C_{2,2} & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}. \quad (1)$$

Given the transmitting nodes in the network, the successful reception events at different receivers are assumed independent. Each transmitter around a given receiver has equal chance for getting successfully received.

The MPR matrix, in general, depends on the channel characteristics, the type of modulation used, and the multiuser detection/equalization method receivers apply. A variety of physical layers has been modeled using MPR and its variants (*e.g.*, CDMA [27], [30], [31] and multiple antenna up-link [32], [33]). Some simple examples of MPR are the *collision channel* $\mathbf{C}_1$ and the *2-collision channel* $\mathbf{C}_2$,

$$\mathbf{C}_1 = \begin{pmatrix} 0 & 1 & & & \\ 1 & 0 & 0 & & \\ 1 & 0 & 0 & 0 & \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}, \mathbf{C}_2 = \begin{pmatrix} 0 & 1 & & & \\ 0 & 0 & 1 & & \\ 1 & 0 & 0 & 0 & \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

As a generalization of $\mathbf{C}_1$ and $\mathbf{C}_2$, we define the *M-collision channel* $\mathbf{C}_M$ in which simultaneous reception of less than or equal to M packets is possible; if more then M packets are transmitted, then none of them are correctly received.

A motivation for considering the M-collision channel comes from the CDMA with matched filter. In the common SINR (signal to interference plus noise ratio) threshold model for CDMA [34], a transmitter is considered successful if its power exceeds noise plus interference (scaled by the spreading gain) with enough margin. If the transmission (and reception) powers of the nodes are identical, then this model is equivalent to the M-collision channel for some M proportional to the spreading gain. Previously, other authors, [25]–[27], considered $\mathbf{C}_M$ in different contexts.

B. Network Capacity

The tools necessary for analyzing the capacity of regular networks are established in Part I. In this section we restate some of these results to reduce the dependence.

Consider a network with N nodes. Rate $\lambda > 0$ is called *uniformly-achievable* if there exists a scheduling and routing policy that delivers packets with rate λ from every node i to every other node j. The *network capacity* (denoted by η) is $(N-1)$ times the maximum of the uniformly achievable rates (multiplication by $(N-1)$ gives the per-node throughput).

We will use the notion of *transport capacity* to upper bound the capacity of networks. The transport capacity, which is introduced in [1], measures the maximum rate-distance product. To handle randomness in receptions, we need to adapt this definition for the MPR model.

Let $l = (i,j)$ denote the link from node i to node j, and $d(l)$ (or, $d(i,j)$) denote the distance between nodes i and j according to some some distance metric $d(\cdot)$. Let $\mathcal{N} = \{1, \cdots, N\}$ be the set of nodes in the network, and $\mathcal{L}$ be the set of links. We denote the packet transmissions by $\mathcal{E} \subset \mathcal{L}$. The transport capacity is defined as

$$\max_{\mathcal{E}\subset\mathcal{L}} \sum_{l\in\mathcal{L}} d(l)\Pi(l;\mathcal{E}), \tag{3}$$

where $\Pi(l;\mathcal{E})$ is the success probability at link l given that the set of transmissions is $\mathcal{E}$. When there is no randomness in receptions, $\Pi(l;\mathcal{E})$ can only be one or zero; in this case, (3) reduces to the summation of distances over the set of successful links. For the MPR model, Thm. 3 of Part I can be restated as follows.

Theorem 1: An upper bound on the network capacity η is given by

$$\eta \le \frac{1}{\bar{L}N} \max_{\mathcal{E}\subset\mathcal{L}} \sum_{l\in\mathcal{L}} d(l)\Pi(l;\mathcal{E}), \tag{4}$$

where $\bar{L}$ is the average distance between two arbitrarily selected nodes, *i.e.*,

$$\bar{L} = \frac{1}{N(N-1)} \sum_{i,j\in\mathcal{N}} d(i,j).$$

The theorem is easier to understand if we view $\sum_{i,j} d(i,j)\lambda$ as the *work* (*i.e.*, the rate-distance product) to be done to uniformly achieve rate λ. Intuitively, the work should be less than the transport capacity of the network,

$$\sum_{i,j} d(i,j)\lambda \le \max_{\mathcal{E}\subset\mathcal{L}} \sum_{l\in\mathcal{L}} d(l)\Pi(l;\mathcal{E})$$

(see [35] for a rigorous proof). This inequality holds for all λ, therefore it holds for the the maximum $\lambda = \eta/(N-1)$; after some algebraic manipulation one gets (4).

In the following sections we will find the capacity of regular networks. Before going into the details, we first outline our methodology. Theorem 1 is our main tool for upper bounding the capacities of regular networks. In order to apply Theorem 1, we will compute the average path length $\bar{L}$ and the transport capacity. After finding an appropriate upper bound on the network capacity, we will show that the upper bound is achieved exactly, or approximately with an error of order $O(1/N)$ (or, with an error $O(1/N^2)$ in ring networks). Specifically, we will find routing and scheduling vectors such that the corresponding RTD (Randomized Time-Division) policy (*c.f.* [35]) achieves the upper bound. The basic idea behind optimal routing in regular networks is to use shortest paths while balancing the routing load. On the other hand, we will see that the optimal MAC problem is equivalent to packing the maximum number of transmissions into a regular lattice.

The RTD policies are introduced and analyzed in Part I. The medium access in RTD relies on probabilistic resource sharing. Namely, in each slot a randomly chosen set of links are activated according to a certain probability distribution (=the so-called *scheduling vector*). The routing in RTD uses a similar idea; each packet is assigned a fixed route randomly according to a distribution over routes (=the *routing vector*). In part I, we established a natural condition for achievability and stability: If the traffic over each link l is less than the average success rate over l, then all packets are eventually delivered to their destinations; the target rates are achieved, and the queues do not blow up. However, we observed that this is true only under a mild condition on the channel reception probabilities—assumption (A1). In Part II, we again assume (A1); how the MPR model relates with (A1) is discussed in Appendix A.

III. Capacity of Manhattan Networks

A node in the Manhattan network is determined by two coordinates $(x,y) \in \{0,\cdots,\sqrt{N}-1\} \times \{0,\cdots,\sqrt{N}-1\}$. We define the distance between two nodes (x_0,y_0) and (x_1,y_1) as the minimum number of hops to reach from one node to another, *i.e.*,

$$d\{(x_0,y_0),(x_1,y_1)\} = \min\{\delta x, \sqrt{N}-\delta x\} + \min\{\delta y, \sqrt{N}-\delta y\}, \tag{6}$$

where $\delta x = |x_0 - x_1|$ and $\delta y = |y_0 - y_1|$. Recall that the nodes on one edge of the Manhattan network are connected to the nodes on the opposite edge; because of this property the distance metric is defined as (6) instead of $d\{(x_0,y_0),(x_1,y_1)\} = \delta x + \delta y$. It can be easily seen that $d\{\cdot\}$ satisfies the triangle inequality. A simple calculation yields the following proposition.

Proposition 1: In the Manhattan network with N nodes, the average distance between two nodes $\bar{L}$ is given by $\sqrt{N}/2 + O(1/\sqrt{N})$, or more precisely,

$$\bar{L} = \begin{cases} \frac{\sqrt{N}}{2}, & \sqrt{N} \text{ odd} \\ \frac{N\sqrt{N}}{2(N-1)}, & \sqrt{N} \text{ even} \end{cases} \tag{7}$$

Proof: See Appendix B. □

The following lemma will be used to show the achievability of the capacity of Manhattan networks.

Lemma 1: In the Manhattan network with uniform traffic λ, there exists a routing vector such that the traffic over any link l is $\lambda(N-1)\bar{L}/4$.

Proof: See Appendix C. □

Lemma 1 is a load balancing property; it guarantees the existence of a routing protocol that distributes the traffic load over the links *uniformly*. In its proof, we show that every *symmetric*, *shortest path* routing satisfies the desired property. Because of using the shortest paths, the quantity in Lemma 1 is the minimum load that has to be put over the links to achieve λ.

Let

$$C_n = \sum_{k=1}^{n} kC_{n,k}$$

denote the expected number of correctly received packets given that n packets are transmitted. The next theorem characterizes the capacity of Manhattan networks.

Theorem 2: (Capacity of Manhattan networks) Let η be the capacity of a Manhattan network of N nodes each with MPR matrix $\mathbf{C}$. Define

$$\eta^* = \max_{i=1,\cdots,4} \frac{C_i}{i+1}\frac{1}{\bar{L}}.$$

The following relations hold:

$$\begin{aligned} \eta &\le \eta^* \\ \eta &= \eta^* + O(\frac{1}{N}). \end{aligned}$$

	$\tau=1$	$\tau=2$	$\tau=3$	$\tau=4$
e_τ	4	3	4	5

TABLE I

e_τ VERSUS τ.

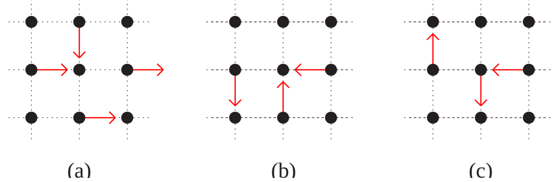

Fig. 3. Figures illustrate the definition of $\mathcal{A}_{ij}$. In (a), the node in the center receives 2 packets intended for itself, and 2 packets intended for other nodes; therefore, it is an element of the set $\mathcal{A}_{4,2}$. The node in the center in (b) receives 2 packets for itself, and 1 packet intended for some other node; it is in $\mathcal{A}_{3,2}$. In (c) the node in the center is not an element of any $\mathcal{A}_{i,j}$ since it is transmitting.

Furthermore, if $\sqrt{N}$ is divisible by e_τ, then $\eta=\eta^*$, where

$$\tau = \arg\max_{i=1,\cdots,4} \frac{C_i}{i+1}, \tag{9}$$

and e_τ is given in Table I.

Proof: First, we will argue that

$$\frac{1}{\bar{L}N}\sum_{l\in\mathcal{L}} \Pi(l;\mathcal{E}) \le \eta^* \tag{10}$$

for every transmission set $\mathcal{E}$. The distance between two neighboring nodes is 1, and the previous inequality proves $\eta \le \eta^*$ as a result of Theorem 1.

To see (10), we will classify the nodes in the network according to the transmission set $\mathcal{E}$. Every node either transmits a packet or stays in the reception mode. Every node k in the reception mode receives two types of packets: the packets intended for the receiver k, and the packets intended for other nodes. Let $\mathcal{A}_{ij}$ be the set of all nodes that do not transmit, receive j packets for itself, and receive $i-j$ packets transmitted for other nodes (see Fig. 3). Define $A_{ij} = |\mathcal{A}_{ij}|$ as the number nodes in $\mathcal{A}_{ij}$. Note that A_{ij} can be non-zero only for $0\le j\le i\le 4$, since nodes can receive packets from at most four other nodes. Every node in the network can transmit one packet at a time and for every receiver in set $\mathcal{A}_{ij}$ there exists j other transmitters in the network. The A_{ij} must satisfy

$$\sum_{i=0}^{4}\sum_{j=0}^{i}(1+j)A_{ij} \le N,$$

since the total number of nodes in the network is N.

At a node receiving j packets for itself and receiving $i-j$ packets intended for other nodes, the expected number of correctly received packets for itself is $\frac{j}{i}C_i$ (see Appendix D). Therefore, the expected number of successful transmissions divided by $\bar{L}N$ is

$$\frac{1}{\bar{L}N}\sum_{l\in\mathcal{L}}\Pi(l;\mathcal{E}) = \frac{1}{\bar{L}N}\sum_{i=0}^{4}\sum_{j=0}^{i}\frac{j}{i}C_iA_{ij}. \tag{12}$$

Consider the optimization problem

$$\begin{aligned} \text{maximize} \quad & \xi = \frac{1}{\bar{L}N}\sum_{i=0}^{4}\sum_{j=0}^{i}\frac{j}{i}C_iA_{ij} \qquad (13) \\ \text{subject to} \quad & \sum_{i=0}^{4}\sum_{j=0}^{i}(1+j)A_{ij} \le N \\ & A_{ij} \ge 0, \end{aligned}$$

where the maximization is with respect to *real* valued A_{ij}. In the original problem, the A_{ij} can only take integer values. Since we relax this constraint (and some others), the solution of the above optimization yields an upper bound on (12).

Optimization in (13) is a linear programming problem, and its solution is well known to be at one of the extreme points of the constraint set. Namely, the solution is attained at

$$A_{ij} = \begin{cases} \frac{N}{j'+1}, & \text{if } i=i',\ j=j' \\ 0, & \text{otherwise,} \end{cases}$$

for some $0\le j'\le i'\le 4$ (the constraint set is a simplex, and these are its corners). When we substitute the possible candidates for A_{ij} in (13), it is seen that

$$\begin{aligned} \xi &= \frac{1}{\bar{L}N}\sum_{i=0}^{4}\sum_{j=0}^{i}\frac{j}{i}C_iA_{ij} \\ &= \frac{1}{\bar{L}}\frac{j'}{i'(j'+1)}C_{i'} \\ &\le \frac{1}{\bar{L}}\frac{C_{i'}}{i'+1} \qquad (15) \\ &\le \eta^*. \end{aligned}$$

The inequality (15) is because $j'/(j'+1)$ is an increasing function of $j'\le i'$. Thus, (10) holds and η^* is an upper bound on the network capacity.

Next, we will show that $\eta^*+O(1/N)$ is achievable. For this, we will use the RTD policy specified by the routing vector in Lemma 1 and a special scheduling vector which will be called *τ-MPR scheduling*. The τ used to achieve $\eta^*+O(1/N)$ is defined by (9). In τ-MPR scheduling, the network is tiled using the τ-MPR pattern (see Fig. 4) and its shifted/rotated versions. In τ-MPR, every scheduled receiver receives τ packets intended for itself. It can be observed that the τ-MPR patterns can tile the network if and only if $\sqrt{N}$ is divisible by e_τ (Table I).

To demonstrate the use of τ-MPR scheduling, suppose that $\tau=1$ (*i.e.*, $\eta^*=C_1/2\bar{L}$) and $\sqrt{N}$ is divisible by 4. In this case, 1-MPR pattern and its shifted/rotated versions (Fig. 5) can tile all of the network. We call each shifted/rotated version of 1-MPR pattern as a *phase* of 1-MPR scheduling. For medium access, we assign $1/8$ probability to each phase in Fig. 5. With this assignment of probabilities each node gets a chance to transmit to each neighbor with probability $1/8$. The probability of success of each transmission is C_1. As a result, every node successfully transmits $C_1/8$ packets on the average to each of its neighbors. Supposing that 1-MPR scheduling is used together with the routing vector provided by Lemma 1, all rates λ satisfying

$$\frac{(N-1)\lambda\bar{L}}{4} \le \frac{C_1}{8} \tag{16}$$

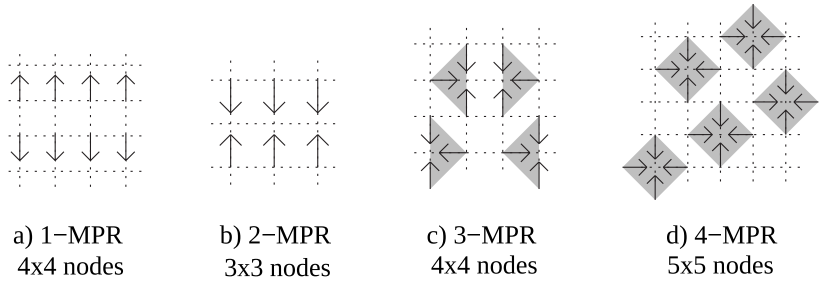

Fig. 4. Different scheduling patterns for τ-MPR, $\tau \in \{1, 2, 3, 4\}$. Dashed lines are the links, and the arrows are scheduled packet transmissions.

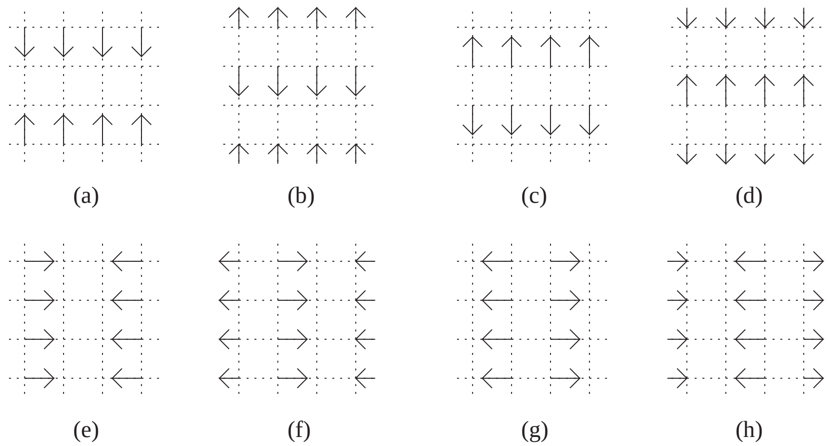

Fig. 5. Eight phases of the 1-MPR scheduling in a network with 16 nodes. In each slot, the medium access protocol applies a randomly selected phase with $1/8$ probability.

are uniformly achievable. This shows that $\lambda = \eta^*/(N-1)$ is uniformly achievable.

Using identical arguments, it is seen that $\lambda = \eta^*/(N-1)\eta^*$ is achieved uniformly by τ-MPR scheduling whenever $\sqrt{N}$ is divisible by e_τ. In case $\sqrt{N}$ is not divisible by e_τ, τ-MPR patterns can tile all of the network but a small portion. In general, the number of nodes which can be scheduled with τ-MPR is $N + O(\sqrt{N})$. Again by using shifted/rotated versions of the τ-MPR,

$$\frac{1}{4}\left[1 + O(\frac{1}{\sqrt{N}})\right] \max_{i=1,\cdots,4} \frac{C_i}{(i+1)} \tag{17}$$

traffic can be supported over each link. The $O(1/\sqrt{N})$ factor in (17) is negative, and it decreases the link capacities. Therefore, in general, η^* may not be achievable, but $\eta^* + O(1/N)$ can be achieved. The achievability part of the theorem follows. □

As a remark, note that τ-MPR is the schedule that maximizes the total number of successful transmissions in the network. This is true exactly if e_τ divides $\sqrt{N}$. Otherwise, τ-MPR provides a good approximation for the schedule that maximizes the total number of successful transmissions. To see this, let's look at the simpler case where the τ-MPR pattern can tile all the network. In τ-MPR, there are $N/(\tau+1)$ receivers in the network each receiving τ packets. Thus, the total number of successful transmissions in the network is $NC_\tau/(\tau+1)$. This quantity divided by $\bar{L}N$ is shown to upper bound the ξ in (15). Hence, τ-MPR maximizes ξ and, equivalently, maximizes the total number of successful transmissions in the network.

In large networks topology discovery may not be feasible, and nodes may not be able use shortest routes. Similarly, during network initialization nodes spend some time discovering the network and may not be able to use the optimal routes. Gossiping [36], flooding, and random walk [37] are alternatives that do not require the nodes to know the whole network topology. In random walk, the packets are relayed at each consecutive hop to a randomly chosen neighbor with uniform probabilities. If the network is connected, every packet eventually reaches its destination, although the delivery may take a long time. In the proof showing the achievability of η we used shortest path routing, which gives average path length $\bar{L}$ proportional to $\sqrt{N}$. In [38], [39], it is shown that in Manhattan networks with random walking packets the average number of relays needed is of the order $N \log N$. Using an argument similar to the one in Theorem 2, it can be easily seen that the maximum achievable rate with random walk routing is $O(\frac{1}{N \log N})$ whereas the capacity scales as $\frac{1}{\sqrt{N}}$. This result shows that the cost of lacking (or not using) the topology information can be very high in large networks.

A. Capacity with Slotted ALOHA

In a distributed wireless network, topology-specific scheduling may not be implementable in practice. On the other hand, it is important to quantify the performance loss due to suboptimal, but easily implementable MAC protocols such as slotted ALOHA. In the next theorem we will give the highest rate achievable with the slotted ALOHA. We consider the capacity setup where every node has infinitely many packets waiting in its queue to be delivered to the other nodes. In slotted ALOHA, every node randomly and independently makes a transmission decision in each slot; a node chooses to transmit a packet with *transmission probability* q, the neighbor to be transmitted is chosen with uniform probabilities. If node i decides to transmit over link $l = (i, j)$, it chooses a route P with probability $Q(P, l)$. [2] Then, if it has a packet with route P, it is transmitted over link l; otherwise, a packet with source i and destination j is transmitted. Symmetric, shortest path routing is assumed.

Theorem 3: (Capacity with slotted ALOHA) The capacity of a Manhattan Network with slotted ALOHA is

$$\eta_{ALOHA} = \frac{1}{4\bar{L}} \max_{0 \le q \le 1} \sum_{k=1}^{4} \binom{4}{k} q^k (1-q)^{5-k} C_k.$$

With the transmission probability

$$q_{max} = \arg\max_{0 \le q \le 1} \sum_{k=1}^{4} \binom{4}{k} q^k (1-q)^{5-k} C_k,$$

every rate $\lambda < \eta_{ALOHA}/(N-1)$ is uniformly achievable with slotted ALOHA. There does not exist any q achieving rates $\lambda > \eta_{ALOHA}/(N-1)$ uniformly.

Proof: Consider an arbitrary link $l = (i, j)$. We will first compute the expected number of successfully transmitted packets over l. In order to receive a packet, node j must stay in the reception mode (this is with probability $1 - q$), and the node i must transmit a packet to node j (this is with probability $q/4$). The probability that $0 \le k' \le 3$ other neighbors of node j transmit is $\binom{3}{k'} q^{k'} (1-q)^{3-k'}$. Given that j does not transmit, i transmits a packet to j, and k' other neighbors transmit, the probability of success over link l is $\tilde{C}_{k'+1,1} := C_{k'+1}/(k'+1)$. ($C_{k'+1}$ is the expected number of successful receptions given $k'+1$ transmissions. Each transmission has equal success probability.) Therefore, the expected number of successfully transmitted packets from i to j is

$$(1-q)\frac{q}{4} \sum_{k'=0}^{3} \binom{3}{k'} q^{k'} (1-q)^{3-k'} \tilde{C}_{k'+1,1}.$$

By setting $k = k' + 1$, we obtain the expected number of successful transmissions as

$$\frac{1}{16} \sum_{k=1}^{4} \binom{4}{k} q^k (1-q)^{5-k} C_k. \qquad (20)$$

Under the slotted ALOHA protocol, the network is identical to the heavy loaded network in Part I. Therefore, if symmetric, shortest path routing is used, then λ satisfying

$$\frac{(N-1)\lambda \bar{L}}{4} \le \frac{1}{16} \sum_{k=1}^{4} \binom{4}{k} q^k (1-q)^{5-k} C_k \qquad (21)$$

[2] $Q(P, l)$ is defined in Sec. 3, [35]. This quantity is chosen to assure fairness, *i.e.*, each route (traffic stream) passing through link l is given bandwidth proportional to its data rate.

	$M = 1$	$M = 2$	$M = 3$	$M = 4$
$\eta \cdot \sqrt{N}$	1.00	1.33	1.50	1.60
$\eta_{ALOHA} \cdot \sqrt{N}$	0.16	0.34	0.46	0.50

TABLE II
CAPACITY VS. SLOTTED ALOHA CAPACITY

is uniformly achievable Maximizing (21) with respect to q, we see that all $\lambda < \eta_{ALOHA}/(N-1)$ are uniformly achievable.

For the converse see Appendix E. □

Omitting an additional $O(1/N)$ factor, rewrite η and η_{ALOHA} as

$$\eta \simeq \frac{1}{\sqrt{N}} \max_{i=1,\cdots,4} \frac{2C_i}{i+1}$$

$$\eta_{ALOHA} \simeq \frac{1}{\sqrt{N}} \max_{0 \le q \le 1} \sum_{k=1}^{4} \binom{4}{k} q^k (1-q)^{5-k} \frac{C_k}{2}.$$

The above expressions show that the scaling law is $O(1/\sqrt{N})$ and the per node throughput goes to zero both with optimal scheduling and slotted ALOHA. This is similar to the capacity law observed in [1]. The main reason behind this fact is the uniform traffic which gives average path length $\bar{L} = O(\sqrt{N})$. Another factor affecting the capacity is the performance of the MAC protocol, which affects the coefficient but not the scaling law. As a numerical example, consider the MPR matrix for M-collision channel, $\mathbf{C}_M$. For $M \in \{1, 2, 3, 4\}$, η and η_{ALOHA} are given in Table II. It is seen that having the best MPR channel C_4 gives only 1.6 times improvement in η over the conventional collision channel C_1. On the other hand, in the collision channel (first column in Table II), using optimal scheduling instead of slotted ALOHA provides about 6 times improvement.

IV. Manhattan Networks with Fading Links

Suppose that each link in the Manhattan network is ON with probability p and OFF with probability $1 - p$. (Here, we mean undirected links; the links (i, j) and (j, i) are always in the same state.) Assume that the network policy does not know which links are ON or OFF, and the nodes transmit their packets *without* knowing if their link is ON or OFF. This will be called a network without link state information (LSI).

Suppose that node i transmits to node j. If the link (i, j) is ON, and if j is the only transmitter in i's neighborhood whose link with i is ON, then the transmission is successful; it is unsuccessful otherwise. This channel can be expressed using an MPR matrix

$$\mathbf{C}_p = \begin{pmatrix} 1-p & p & & & \\ 1-2p(1-p) & 2p(1-p) & 0 & & \\ 1-3p(1-p)^2 & 3p(1-p)^2 & 0 & 0 & \\ 1-4p(1-p)^3 & 4p(1-p)^3 & 0 & 0 & 0 \end{pmatrix}.$$

In the MPR matrix, the entry $C_{k,1}$ is the probability that k neighbors transmit and one of them gets through, which is the case only when one link is ON and the rest $k - 1$ are OFF; the probability of this event is $\binom{k}{1} p (1-p)^{k-1}$.

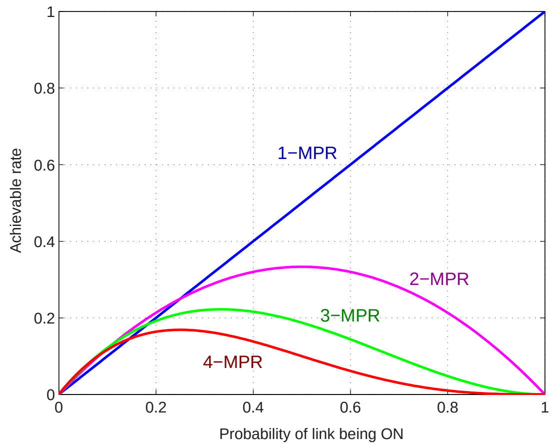

Fig. 6. τ-MPR curve shows $\frac{\tau(1-p)^{\tau-1}}{\tau+1}2p$. This is $\sqrt{N}$ times the rate achievable with τ-MPR scheduling in a Manhattan network with fading links. Upper envelope of these curves is $\eta\cdot\sqrt{N}$. The performance of the scheduling patterns is opposite for small p, *i.e.*, 4-MPR gives the highest throughput and 1-MPR gives the lowest.

For this channel, Theorem 2 gives the network capacity as

$$\eta \simeq \frac{1}{\sqrt{N}} \max_{i=1,\cdots,4} \frac{i(1-p)^{i-1}}{(i+1)} 2p.$$

Theorem 2 also gives a way to schedule packets optimally. The value of

$$\begin{aligned} \tau &= \arg\max_{i=1,\cdots,4} \frac{i(1-p)^{i-1}}{(i+1)} \\ &= \begin{cases} 1 & 1 \geq p \geq \frac{1}{4} \\ 2 & \frac{1}{4} \geq p \geq \frac{1}{9} \\ 3 & \frac{1}{9} \geq p \geq \frac{1}{16} \\ 4 & \frac{1}{16} \geq p \geq 0 \end{cases} \end{aligned} \quad (22)$$

determines which τ-MPR pattern (Fig. 4) to use as a function of the severity of fading. From (22), it is apparent that one should use higher τ's when p is smaller. Using higher τ for small p can be interpreted as multiuser diversity. For instance when p is very small, in the neighborhood of receiver it is a very small probability that there is more than a single link ON. Therefore, 4-MPR scheduling (namely, "all neighbors transmit to the node in the center" strategy) increases the probability of successful transmission, and does not lead to frequent collisions. The rates achievable with τ-MPR scheduling, $\tau \in \{1,2,3,4\}$, are shown in Fig. 6.

It is an interesting question to ask what would be the improvement due to having and exploiting the LSI. In case of LSI, the optimal policy again follows a similar idea: given the fading configuration, find and use the transmission schedule that maximizes the number of successful transmissions. However, in this case it is very hard to compute the achievable rates since there are numerous fading configurations. The following theorem gives some bounds on the capacity with LSI.

Theorem 4: (Capacity of Manhattan networks with LSI) Let $\eta^{\#}$ be the capacity of the Manhattan network with LSI. Then,

$$1 \leq \frac{\eta^{\#}}{\eta} \leq 2.86 + O(1/\sqrt{N}). \quad (23)$$

Moreover,

$$\lim_{\substack{N\to\infty \\ p\to 0}} \frac{\eta^{\#}}{\eta} = 2.5 \quad \text{and} \quad \lim_{\substack{N\to\infty \\ p\to 1}} \frac{\eta^{\#}}{\eta} = 1. \quad (24)$$

Proof: We will first discuss the extreme cases $p \simeq 0,\ p \simeq 1$. In these two regimes the results are easy to understand since there are simple strategies with performance close to the optimal.

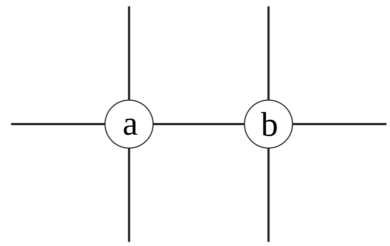

Fig. 7. Scheduling example with LSI

If $p \simeq 0$, then very few links are ON, and the optimal strategy is transmitting over almost every ON. We describe a strategy that will be called *all ON's scheduled* next. Let a and b be two nodes in the network (Fig. 7). In every slot, schedule a transmission over link (a,b) if and only if the link between a and b is ON, and all of the other six links connecting a and b to their respective neighbors are OFF. Choose the direction of transmission randomly; a to b with probability $1/2$, and b to a with probability $1/2$. With this scheduling the traffic that can be carried in each direction is $p(1-p)^6/2$ (this is the probability that the link is scheduled in a direction). Using a symmetric, shortest path routing we see that rates less than $2p(1-p)^6/(N-1)\bar{L}$ are uniformly achievable. Furthermore, rates above $2p/(N-1)\bar{L}$ are not uniformly achievable. This is true, since there are total $2N$ (undirected) links in the network and the expected number of ON links is $2pN$. Therefore, the transport capacity averaged over the link states is upper bounded by $2pN$. As it is done in Part I, Theorem 1 can be extended to include networks with multiple states by replacing the transport capacity in the upper bound by the *average* transport capacity. Therefore, $\eta^{\#} \leq 2p/\bar{L}$ follows.

We have just shown that

$$\frac{2p(1-p)^6}{\bar{L}} \leq \eta^{\#} \leq \frac{2p}{\bar{L}}. \quad (25)$$

When all sides are divided by

$$\eta = \frac{4p(1-p)^3}{5\bar{L}} \left[1 + O(\frac{1}{\sqrt{N}})\right],$$

we get

$$\frac{5(1-p)^3}{2} \left[1 + O(\frac{1}{\sqrt{N}})\right] \leq \frac{\eta^{\#}}{\eta} \leq \frac{5}{2(1-p)^3} \left[1 + O(\frac{1}{\sqrt{N}})\right].$$

The left hand side of (24) follows when we take the limit $p\to 0, N\to\infty$.

"All ON's scheduled" strategy almost achieves the capacity with LSI, which is $\eta^{\#} \simeq \frac{2p}{\bar{L}}$ for $p \simeq 0$. However, without LSI, the optimal strategy is 4-MPR scheduling which uses only $2/5$ of the available links (this fact can be seen by counting the number of used links in Fig. 4.d). This gives the "all ON's scheduled" strategy an advantage of 2.5 times over 4-MPR scheduling.

Next, we will look at the regime $p \simeq 1$. Note that the capacity with LSI is always less than the capacity without fading, *i.e.*, $\eta^{\#} \leq 1/2\bar{L}$. Moreover, the capacity with LSI is greater than capacity without LSI, $\eta \leq \eta^{\#}$. Hence, the following holds

$$\frac{p}{2\bar{L}}\left[1+O(\frac{1}{\sqrt{N}})\right] = \eta \leq \eta^{\#} \leq \frac{1}{2\bar{L}}$$

for $p > 1/4$. Divide all sides by η,

$$1 \leq \frac{\eta^{\#}}{\eta} \leq \frac{1}{p}\left[1+O(\frac{1}{\sqrt{N}})\right].$$

Taking the limit $p\to 1, N\to\infty$ gives the right hand side of (24). One conclusion is that if $p \simeq 1$ then almost all links are always ON, and with LSI using $1-MPR$ is almost optimal. The results for $p \simeq 0,\ p \simeq 1$ also suggest that the knowledge of LSI is more valuable when p is small. For high values of p, LSI is less important; one can simply use 1-MPR scheduling.

Next, we will upper bound the transport capacity for an arbitrary p. Let $\mathcal{E}_v$ be the schedule maximizing the number of successful receptions when the network is in state v. (The network state v denotes the state of every link; in accordance with Part I, we denote the probability density over the network states by $p(v)$.) Without loss of generality assume that each link l in the schedule $\mathcal{E}_v$ is successful. Hence, the transport capacity averaged over the set of states is equal to

$$\sum_v p(v) \sum_{l\in\mathcal{L}} 1(l \in \mathcal{E}_v).$$

Observe that $\sum_{l\in\mathcal{L}} 1(l\in\mathcal{E}_v)$ is equal to

$$\frac{1}{2}\sum_{i\in\mathcal{N}} 1((i,j)\in\mathcal{E}_v \text{ or } (j,i)\in\mathcal{E}_v \text{ for some } j),$$

where the factor $1/2$ comes from the fact that each successfully transmitted packet is counted twice; once at the transmitter, and another time at the receiver. So, we can write the average transport capacity as

$$\begin{aligned}&\sum_v p(v)\frac{1}{2}\sum_{i\in\mathcal{N}} 1((i,j) \text{ or } (j,i)\in\mathcal{E}_v \text{ for some } j)\\ &= \frac{1}{2}\sum_{i\in\mathcal{N}}\sum_v p(v) 1((i,j) \text{ or } (j,i)\in\mathcal{E}_v \text{ for some } j).\end{aligned}$$

The final sum $\sum_v p(v)1((i,j)$ or $(j,i) \in \mathcal{E}_v$ for some $j)$ is nothing but the expected number of successfully transmitted or received packets by node i. Since every node has four neighbors, this expectation is less than or equal to $1-(1-p)^4$, which is the probability that at least one out of four links is ON. Thus, we have proved the following upper bound

$$\begin{aligned}\sum_v p(v)\sum_{l\in\mathcal{L}} 1(l\in\mathcal{E}_v) &\leq \frac{1}{2}\sum_{i\in\mathcal{N}}(1-(1-p)^4)\\ &= \frac{N}{2}(1-(1-p)^4).\end{aligned}$$

As before, the extension of Theorem 1 with the average transport capacity gives $\eta^{\#} \leq (1-(1-p)^4)/2\bar{L}$. When we divide by η,

$$\begin{aligned}\frac{\eta^{\#}}{\eta} &= \frac{\eta^{\#}}{\eta^*(1+O(1/\sqrt{N}))}\\ &\leq \left[\max_{i=1,\cdots,4}\frac{i(1-p)^{i-1}}{(i+1)}\right]^{-1}\frac{1-(1-p)^4}{2p}+O(\frac{1}{\sqrt{N}}).\end{aligned}$$

The last quantity is upper bounded by $2.853.. + O(1/\sqrt{N})$, which is achieved at $p = 0.1111...$ Therefore, (23) follows. □

V. Other Regular Topologies and Optimal Connectivity

Consider a Manhattan network with 2-hop connectivity, *i.e.*, every node is connected to neighbors two hops or one hop away (there are twelve such neighbors, see Fig. 8.a). Consider the scheduling pattern in Fig. 8.b. This pattern can be used under the assumption that the nodes can perfectly receive 8 packets simultaneously (*i.e.*, the MPR matrix is $\mathbf{C}_8$—this channel can be viewed as an abstraction for CDMA with high spreading gain). When we tile the network with such a pattern, approximately 8/13 of the nodes are transmitters and the 1/13 nodes are receivers. Each transmitted packet moves 2 distance units, and the expected progress (rate-distance product) with this scheduling is

$$\frac{16}{13}N + O(\sqrt{N}).$$

This quantity is higher than the expected progress with the 4-MPR pattern in a 1-hop connected network (*i.e.*, the network considered in previous sections). In 4-MPR scheduling, approximately $1/5$ of the nodes are scheduled as receivers and each receiver gets 4 packets moving a single distance unit. Hence, the expected progress with 4-MPR scheduling is

$$\frac{4}{5}N + O(\sqrt{N}).$$

Here, we see that multipacket receptions, which is the case in CDMA, can not be exploited sufficiently if the connectivity is not high enough. In the next theorem, we argue that the increase in expected progress translates to increased achievable rates.

Theorem 5: (Capacity of the 2-hop connected network) Let η_{2-HOP} be the capacity of a 2-hop connected Manhattan network with MPR matrix $\mathbf{C}_8$. Then,

$$\frac{16}{13}\frac{1}{\bar{L}} + O(\frac{1}{N}) \leq \eta_{2-HOP} \leq \frac{16}{9}\frac{1}{\bar{L}} \qquad (28)$$

The lower bound is about 54% higher than $\eta = \frac{4}{5}\frac{1}{\bar{L}} + O(\frac{1}{N})$, the capacity of a 1-hop connected Manhattan network with $\mathbf{C}_8$.[3]

Proof: The achievability of the lower bound is essentially same as the achievability of η in 1-hop connected Manhattan networks. The differences and similarities are the following. In a 2-hop connected network, it is advantageous for packets to move 2 hops per transmission rather than 1-hop. The routing

[3] The lower bound in (28) is achievable for every $\mathbf{C}_M,\ M \geq 8$. Hence, this statement is true for any such $\mathbf{C}_M$.

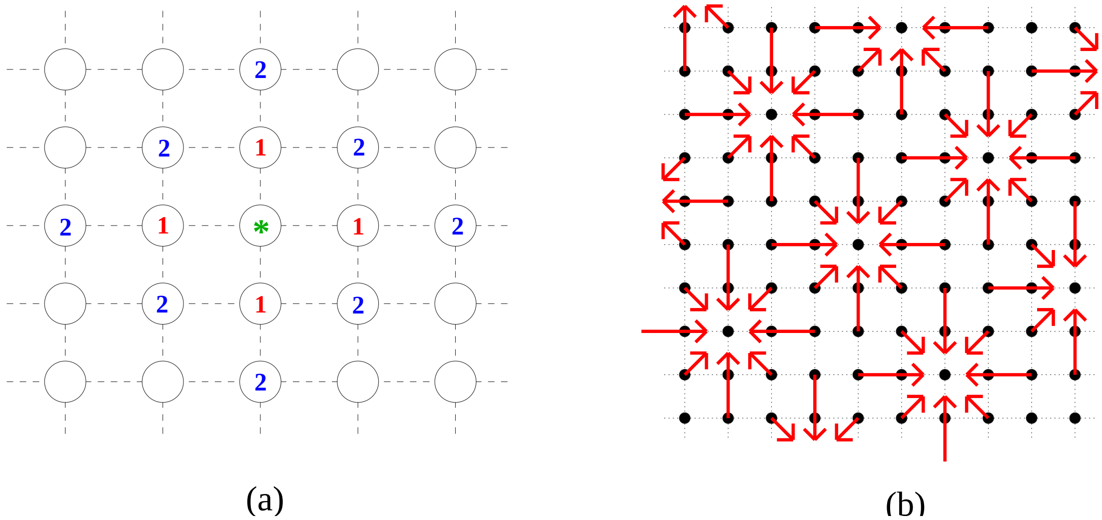

Fig. 8. (a) The neighbors of in the 2-hop connected Manhattan network are shown. Consider the node in the center (marked with $*$). It has total 12 neighbors, four of them are 1-hop neighbors (marked with 1), and eight are 2-hop neighbors (marked with 2). (b) 8-MPR scheduling in a Manhattan network with two hop connectivity. The network is divided into groups of 13 nodes. In each group the node in the center is the receiver, the receiver's two-hop neighbors are transmitters, and the 1-hop neighbors of each receiver stay idle.

vector achieving the lower bound uses this idea; packets keep jumping 2 hops until they reach to the destination, or to a 1-hop neighbor of the destination after which they have a single hop to go. The traffic load over links can be balanced in a way similar to Lemma 1. Moreover, in a large network the traffic for the 1-hop neighbors is negligible compared to traffic for the 2-hop neighbors. In a load balanced network the 8-MPR pattern and its shifted versions achieve the lower bound.

Next, we will prove the upper bound. Consider a transmission schedule $\mathcal{E}$. Let k be a node hearing hearing j_r transmissions from its r-hop neighbors, $r = 1, 2$, intended for itself. Let A_{j_1,j_2} be the number of all such nodes in $\mathcal{E}$. Observe that j_1, j_2 must lie in

$$\mathcal{J} = \{(j_1, j_2) \in \mathcal{Z}_+^2 : j_1 \le 4, j_1 + j_2 \le 8\}$$

(this is under the assumption that every transmitter is successful; otherwise, we can eliminate some unnecessary transmissions from $\mathcal{E}$ without affecting the successful ones). A_{j_1,j_2} must satisfy

$$\sum_{(j_1,j_2)\in\mathcal{J}} A_{j_1,j_2}(1 + j_1 + j_2) \le N, \tag{29}$$

since the total number of nodes is less than N. Further, observe that

$$\frac{1}{\bar{L}N} \sum_{l\in\mathcal{L}} d(l)\Pi(l;\mathcal{E}) = \frac{1}{\bar{L}N} \sum_{(j_1,j_2)\in\mathcal{J}} A_{j_1,j_2}(j_1 + 2j_2). \tag{30}$$

Maximizing (30) under the constraint (29), and A_{ij} being non-negative and real gives

$$\frac{1}{\bar{L}N} \sum_{l\in\mathcal{L}} d(l)\Pi(l;\mathcal{E}) \le \frac{1}{\bar{L}N} \max_{(j_1,j_2)\in\mathcal{J}} \left\{ N \frac{j_1 + 2j_2}{1 + j_1 + j_2} \right\} = \frac{16}{9}\frac{1}{\bar{L}}.$$

We get the upper bound by invoking Theorem 1. □

The previous theorem bounds the capacity for a *given* MPR matrix. Namely, it addresses the question "Given the physical layer, what is the best MAC/routing/connectivity?" For example, this question is relevant to CDMA networks, where multi-packet reception already exists. To find the ultimate network capacity, one ought to optimize with respect to the physical layer, too. However, we do not attempt this optimization in this paper.

Next, we will consider the ring networks. Assume that the nodes are uniformly placed on a ring with unit circumference (see Fig. 9). The distance between node i and node j is given by $d(i, j) = \min\{\delta, 1-\delta\}$, where $\delta = |i-j|/N$. We will consider the following simple reception model which is an extension of the collision channel: If $\{(i_k, j_k) : k = 1, 2, \cdots, K\}$ is a set of transmitter-receiver pairs then the transmission from i_k to j_k is successful if

$$d(i_r, j_k) > d(i_r, j_r) \quad \text{for all } r \ne k.^4$$

The rationale behind this model is that each transmitter i_r uses just sufficient power to reach its destination j_r; this transmission causes interference to all nodes within $d(i_r, j_r)$ neighborhood of the transmitter, and negligible interference to the nodes outside. Transmission from i_k to j_k is successful only if j_k is sufficiently far apart from every other transmitter.

Lemma 2: Suppose that each transmission (i, j) is represented with an arrow from i to j with length $d(i, j)$. Then, the arrows corresponding to two successful transmissions do not intersect.

Proof: The proof is by contradiction. Consider two successful transmissions $(i_1, j_1), (i_2, j_2)$. If j_2 were between i_1 and j_1, then (i_2, j_2) could not be successful. Similarly, if i_2 were between i_1 and j_1, and j_2 were outside, then the (i_2, j_2) arrow would either go over j_1 (which makes (i_1, j_1) unsuccessful), or would go over i_1. In the latter case, to make (i_2, j_2) successful j_2 must be at least $d(i_1, j_1)$ apart from i_1. However,

[4] The results in this section can be generalized with minor changes to the model $d(i_r, j_k) > (1 + \Delta)d(i_r, j_r)$ for some $\Delta > 0$.

if j_2 is placed that apart, then the interference from i_2 makes (i_1, j_1) unsuccessful. □

The previous lemma says that the premium quantity in the ring is space. The summation of the lengths of the arrows can not exceed the circumference of the ring. Therefore, the transport capacity of the ring is upper bounded by 1.

Let's see how close we can approach to the transport capacity. We will consider two cases: 1-hop connected ring (nodes are restricted to communicate with their nearest two neighbors) and the ring without constraint. An important observation is that in the 1-hop connected ring we can use a one-dimensional analogue of 1-MPR scheduling; see the so-called 1-*RING scheduling* in Fig. 9.b. The 1-RING pattern can be used to cover all ring, and it uses *half* of the space available. That is, when the number of nodes is even, half of the nodes get a chance to transmit and the distance of each transmission is $1/N$. Therefore, the expected progress (rate-distance product) is $1/2$. Similarly, the expected progress of 1-RING scheduling for odd N can be obtained as $1/2 + O(1/N)$.

In the ring without constraint there are other possibilities for communication. In particular, consider the so-called τ-*RING scheduling* in which every packet jumps τ hops (see the 4-RING example in Fig. 9.c). In τ-RING the transmitter-receiver pairs are given by

$$(0,\tau),(2\tau+1,\tau+1),(2(\tau+1),2(\tau+1)+\tau),\\ (3(\tau+1)+\tau,3(\tau+1)),\cdots$$

where the list is truncated at the point either a transmitter or a receiver index goes above $N-1$. If the τ-RING pattern perfectly covers the network (*i.e.,* N is divisible by $\tau+1$), then $N/(\tau+1)$ transmissions each over τ-hops are scheduled. This means the expected progress of τ-RING is $\tau/(\tau+1)$. For general N τ-RING may not cover the network, but the expected progress is $\tau/(\tau+1)+O(1/N)$.

From these observations we see that the expected progress is an increasing function of τ, and the 1-RING scheduling gives the minimum expected progress. In the limit $\tau\to\infty$, we get $\tau/(\tau+1)\to 1$. Since the transport capacity can not be larger than 1, we see that the τ-RING patterns achieve the transport capacity in the limit. The following theorem gives analogous results for the network capacity.

Theorem 6: (Capacity of the Ring) The capacity of 1-hop connected ring is

$$\eta_{1-RING} = \frac{2}{N} + O(\frac{1}{N^2}). \tag{32}$$

On the other hand, the capacity of the unconstrained ring satisfies

$$\frac{4\tau}{\tau+1}\frac{1}{N} + O(\frac{1}{N^2}) \le \eta_{RING} \le \frac{4}{N}, \tag{33}$$

for every $\tau = 1, 2, \cdots$. Therefore, the capacity of the unconstrained ring is double that of the minimally connected ring.

Proof: 1-RING scheduling and its shifted versions achieve (32). The general τ-RING, $\tau \ge 1$ achieves the lower bound in (33). The routing uses shortest paths; packets hop τ hops until they reach the τ neighborhood of their destination. The shortest path routing, as before, balances the traffic. Further, for large N, it can be seen that only a negligible fraction of the traffic is diverted to neighbors less than τ hops away (for such packets $\tau' < \tau$ RING scheduling is used). The main ideas are same as the ones in Theorems 2 and 5, and we will not go through the details to avoid repetition.

The average path length in the ring satisfies $\bar{L} \ge 1/4$. Furthermore,

$$\bar{L} = 1/4 + O(1/N).$$

We will do this computation only for odd N; the other case is almost identical.

$$\begin{aligned}
\bar{L} &= \frac{1}{N-1}\sum_{i=1}^{N-1} d(0,i) \\
&= \frac{1}{(N-1)}\sum_{i=-(N-1)/2}^{(N-1)/2}\frac{|i|}{N} \\
&= \frac{2}{N(N-1)}\sum_{i=0}^{(N-1)/2} i \\
&= \frac{2}{N(N-1)}\frac{N^2-1}{8} \\
&= \frac{N+1}{4N} \\
&= \frac{1}{4} + O(\frac{1}{N}).
\end{aligned}$$

In the unconstrained ring, the transport capacity is upper bounded by 1 as argued after Lemma 2. Therefore, the capacity is $\le 1/(N\bar{L})$ by Theorem 1. Since $\bar{L} \ge 1/4$, we get the upper bound $\eta_{RING} \le 4/N$.

In the minimally connected ring, we need to prove that the transport capacity is upper bounded by $1/2 + O(1/N)$ under the restriction that nodes can only talk to immediate neighbors; this can be easily done using a technique identical to the ones in Theorems 2 and 5. The upper bound for η_{1-RING} follows. □

VI. Conclusion

In Part II, we applied the RTD policies and the flow analysis to the ring and Manhattan networks. We obtained a closed-form expression for the capacity of Manhattan networks and analyzed the impact of link fading, link state information and the topology information on achievable rates. We also compared a suboptimal scheme that uses ALOHA as its medium access to the optimal policy that jointly optimize medium access and routing. We finally examined the effect of variable connectivity on the capacity of Manhattan and ring networks.

The results for regular networks have ramifications for MAC in arbitrary networks. In Manhattan networks with multipacket receiving nodes the τ-MPR patterns (Figs. 4 and 8.b), namely "neighboring nodes transmit into the center" strategy, was shown to be optimal for medium access. The τ-MPR scheduling locally resembles to an up-link especially for τ large. We expect this type of scheduling to be useful in arbitrary networks where multipacket reception is possible with multiple receive antennas or spread-spectrum. The 1-MPR scheduling and τ-RING scheduling (Figs. 4 and 9), namely "transmitters turn

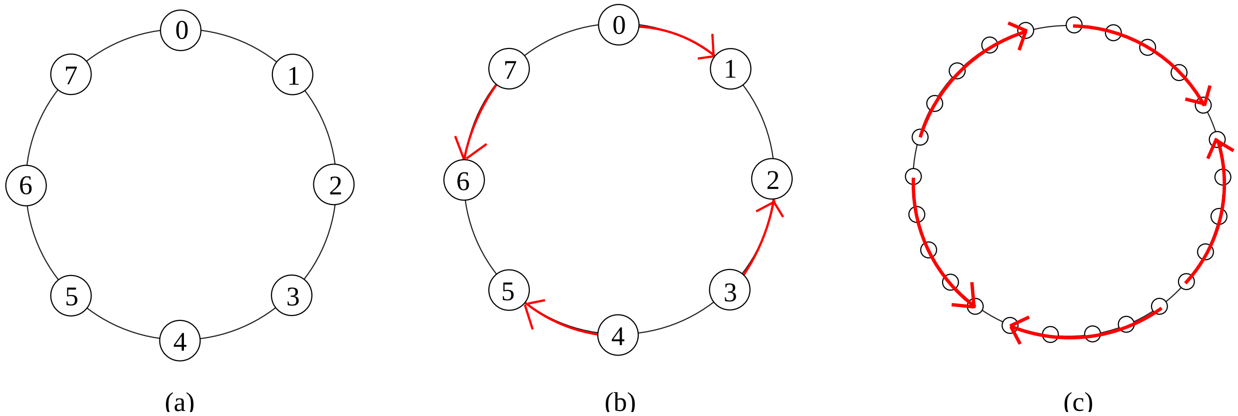

Fig. 9. (a) A ring network with 8 nodes. The circumference is of unit length, and the distance between any two neighbors is 1/8 units. (b) 1-RING scheduling in a ring with 8 nodes. (c) 4-RING scheduling in a ring with 25 nodes.

each other their back and transmit" strategy, were shown to be optimal in networks without multipacket reception. We expect this idea to be useful in wireless networks with parts locally resembling to a one-dimensional topology. Examples include a wireless LAN in a corridor, or a group of nodes on a street or a highway.

There is an important open question related to the capacity of arbitrary networks: Does our analysis for regular networks carry over to arbitrary networks? In regular networks it is shown that the transport capacity provides tight upper bounds on the capacity. This upper bound is not always achievable in arbitrary networks, however, it suggests a general duality relation between the transport capacity and network capacity. To see what we mean by duality, recall that the transport capacity upper bound, inequality (4), is valid for all distance metrics satisfying the triangle inequality. Hence, the upper bound minimized over all distance metrics is still an upper bound. But, how does the minimized upper bound compare with the network capacity? Under what conditions is the minimized upper bound achievable?[5] These are some questions which seem to deserve further attention.

APPENDIX

A. Assumption (A1) for the MPR Model

In part I, we characterized the capacity and the stability regions under a mild condition—assumption (A1). The MPR model is a special case of the model in Part I, and we need an equivalent condition. Let $\mathcal{E} = \{1, 2, \cdots, n\}$ be the set of transmitting nodes neighboring a receiver, $\{i_1, i_2, \cdots, i_k\}$ be any k-element subset of $\{1, 2, \cdots, n\}$, and $\mathcal{F}$ be the set of correctly received packets by the receiver. Note that the number of k-element subsets of $\mathcal{E}$ is $\binom{n}{k}$, and an assumption of the MPR model is that the reception event of every k-element subset is with equal probability. Then,

$$\Pr\{\mathcal{F} = \{i_1, i_2, \cdots, i_k\} \mid \mathcal{E} = \{1, 2, \cdots, n\}\} = \frac{C_{n,k}}{\binom{n}{k}}.$$

[5] An anonymous reviewer pointed out that for a given distance metric the upper bound is achievable if only if the traffic requirements is such that the network can continuously operate according to the schedule(s) that attain the maximum in (3).

Define

$$\tilde{C}_{n,k} = \Pr\{\{i_1, i_2, \cdots, i_k\} \subset \mathcal{F} \mid \mathcal{E} = \{1, 2, \cdots, n\}\},$$

which is the marginal probability of success for the transmitters $\{i_1, i_2, \cdots, i_k\}$. A simple counting argument shows that

$$\tilde{C}_{n,k} = \sum_{m=k}^{n} \binom{n-k}{m-k} \frac{C_{n,m}}{\binom{n}{m}}.$$

(A1) We require the marginal probability of success to be lower when there are more transmissions, *i.e.*,

$$\tilde{C}_{n_1,k} \geq \tilde{C}_{n_2,k} \tag{37}$$

for all $k \leq n_1 \leq n_2$.

This condition is equivalent to (A1) in Part I, and it eliminates MPR matrices such as

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & & \\ 0 & 0 & 1 & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix},$$

which is not encountered in practice.

B. Proof of Proposition 1

The average distance a packet originating from node i travels is same for all i. As a result of this symmetry, we can compute $\bar{L}$ by averaging the distances between the node (0,0) and the other nodes in the network,

$$\begin{aligned} \bar{L} &= \frac{1}{N-1} \sum_{x=0}^{\sqrt{N}-1} \sum_{y=0}^{\sqrt{N}-1} d\{(0,0),(x,y)\} \\ &= \frac{1}{N-1} \sum_{x=0}^{\sqrt{N}-1} \sum_{y=0}^{\sqrt{N}-1} \min\{x, \sqrt{N}-x\} + \min\{y, \sqrt{N}-y\} \\ &= \frac{2\sqrt{N}}{N-1} \sum_{x=1}^{\sqrt{N}-1} \min\{x, \sqrt{N}-x\}. \end{aligned}$$

The two cases follow from the last expression.

C. Proof of Lemma 1

In the following we will only consider links l connecting two neighboring nodes, and paths $P = (l_1, l_2, \cdots, l_k)$ composed of such links. If $P = (l_1, l_2, \cdots, l_k)$, we say that the number of links on path P is k, and write $|P| = k$. We denote the set of (non-cyclic) paths from i to j by $\mathscr{P}_{ij}$ and $\mathscr{P} := \cup_{i,j}\mathscr{P}_{ij}$. A routing vector $H = (x_P \geq 0 : P \in \mathscr{P})$ shows the traffic flow in the network. For some $P \in \mathscr{P}_{ij}$, x_P is the fraction of traffic following path P (we require $\sum_{P\in\mathscr{P}_{ij}} x_P = 1$ for all $i \neq j$). A routing vector $H = (x_P \geq 0 : P \in \mathscr{P})$ is called a *shortest path* routing vector if $x_P > 0,\ P \in \mathscr{P}_{ij}$ implies $|P| = d(i, j)$.

For a given source destination pair there exists many routes with the minimum path length. It is the objective of a routing vector to use shortest distances while distributing the load uniformly among links. In this appendix, we will prove that all *symmetric*, shortest path routing vectors satisfy

$$\sum_{i,j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{ij}:\, l\in P} x_P = \frac{(N-1)\bar{L}}{4} \tag{39}$$

(the left hand side is the traffic over link l for $\lambda = 1$; (39) suffices to prove the lemma). Before going into the details of what we mean by symmetry, it is useful to give an example. Consider a source-destination pair $i, j \in \mathcal{N}$. Define

$$\mathscr{P}'_{ij} = \{P \in \mathscr{P}_{ij} : |P| = d(i,j)\},$$

and

$$x_P = \begin{cases} \frac{1}{|\mathscr{P}'_{ij}|}, & \text{if } P \in \mathscr{P}'_{ij} \\ 0, & \text{otherwise.} \end{cases}$$

The vector $H = (x_P : P \in \mathscr{P})$ is symmetric, and it distributes the routing load uniformly over all links with minimum path length.

We will use modulo arithmetic for discussing translation of nodes, links and paths. $(x, y) \in \mathcal{Z}^2$ refers to the node $(\text{mod}(x), \text{mod}(y))$ where $\text{mod}(x) = x \mod \sqrt{N}$ is the usual modulo function. When node $i = (x, y)$ is shifted by $\delta = (\delta(x), \delta(y))$, node

$$i + \delta = (x + \delta(x), y + \delta(y))$$

is obtained. Define the δ translation of link $l = (i, j)$ as $l + \delta = (i + \delta, j + \delta)$, and the δ translation of path $P = (l_1, l_2, \cdots, l_k)$ as

$$P + \delta = (l_1 + \delta, l_2 + \delta, \cdots, l_k + \delta).$$

We call a routing vector *shift invariant* if for all $i, j \in \mathcal{N}, P \in \mathscr{P}_{ij}, \delta \in \mathcal{Z}^2$,

$$x_P = x_{P+\delta}$$

is satisfied.

Denote the origin node with 0. Next, we will argue that if a routing vector is shift invariant then for every link l,

$$\sum_{i,j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{ij}:\, l\in P} x_P = \sum_{j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{0j}} x_P \psi(P, l), \tag{40}$$

where

$$\psi(P, l) = \sum_{i\in\mathcal{N}} 1(l \in P + i). \tag{41}$$

To see (40), make a change of variables $j' = j - i$,

$$\begin{aligned} &\sum_{i,j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{ij}} x_P 1(l \in P) \\ &\quad= \sum_{j'\in\mathcal{N}} \sum_{i\in\mathcal{N}} \sum_{P\in\mathscr{P}_{i,j'+i}} x_P 1(l \in P) \\ &\quad= \sum_{j'\in\mathcal{N}} \sum_{P\in\mathscr{P}_{0,j'}} \sum_{i\in\mathcal{N}} x_{P+i} 1(l \in P + i) \\ &\quad= \sum_{j'\in\mathcal{N}} \sum_{P\in\mathscr{P}_{0,j'}} x_P \sum_{i\in\mathcal{N}} 1(l \in P + i). \end{aligned}$$

Last equality is due to the translation invariance of the routing vector.

The links in a Manhattan network are in four directions: up, down, left, right. The number $\psi(P, l)$ is the number of links in P which are in the same direction with l. Definition (41) assures that $\psi(P, l)$ depends only the direction of l but not its location, ***i.e.***, $\psi(P, l) = \psi(P, l + \delta)$ for every δ.

Eqn. (40) can be interpreted as follows. Suppose that the network traffic is uniform and equal to 1 for every source destination pair. Left hand side in (40) is the routing load over link l. (40) implies that the routing load on each link l depends only on the direction of l. Moreover, (40) suggests an alternative view of computing routing load over link l: Fix the origin as the source node, and add up the traffic from the origin to the other nodes, passing through links in the same direction with l.

In order to define a symmetric routing vector, we need a few other definitions. Let the vertical reflection of node $i = (x, y)$ be $i_\updownarrow = (x, -y)$, the horizontal reflection be $i_\leftrightarrow = (-x, y)$, and the rotation be $i_\circlearrowleft = (-y, x)$. Similarly, let vertical reflection of a link $l = (i, j)$ be $l_\updownarrow = (i_\updownarrow, j_\updownarrow)$, and vertical reflection of a path $P = (l_1, l_2, \cdots, l_k)$ be

$$P_\updownarrow = (l_{1\updownarrow}, l_{2\updownarrow}, \cdots, l_{k\updownarrow}).$$

Horizontal reflection and rotations of links $l_\leftrightarrow,\ l_\circlearrowleft$ and paths $P_\leftrightarrow,\ P_\circlearrowleft$ are defined similarly. Call a translation invariant routing vector *symmetric* if for all $P \in \mathscr{P}_{0j}$,

$$x_P = x_{P_\updownarrow} = x_{P_\leftrightarrow} = x_{P_\circlearrowleft} \tag{42}$$

is satisfied.

From the definition of ψ, it follows that for all $* \in \{\updownarrow, \leftrightarrow, \circlearrowleft$ },

$$\psi(P, l) = \psi(P_*, l_*). \tag{43}$$

Therefore, if a routing vector is symmetric then,

$$\begin{aligned} \sum_{j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{0j}} x_P \psi(P, l) &= \sum_{j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{0j}} x_{P_*} \psi(P_*, l_*) \\ &= \sum_{j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{0j}} x_P \psi(P, l_*). \end{aligned} \tag{44}$$

The first equality follows from (42) and (43). The second equality is due to the fact that if we map every $P \in \mathscr{P}_{0j}$ to P_* then we again obtain the set $\mathscr{P}_{0j}$.

Let l be a link pointing up. Then, $l_\circlearrowleft, l_\updownarrow, (l_\circlearrowleft)_\leftrightarrow$ are vectors pointing left,down and right, respectively. If the routing vector is symmetric, then (40) and (44) ensure that the traffic on links $l, l_{\circlearrowleft}, l_{\updownarrow}, (l_{\circlearrowleft})_{\leftrightarrow}$ are the same.

Let $H = (x_P : P \in \mathscr{P})$ be a symmetric, shortest path routing vector. Next, we will prove that H satisfies (39). From definitions, it follows that

$$|P| = \psi(P, l) + \psi(P, l_{\circlearrowleft}) + \psi(P, l_{\updownarrow}) + \psi(P, (l_{\circlearrowleft})_{\leftrightarrow}). \quad (45)$$

Therefore,

$$\begin{aligned}
\sum_{i,j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{ij}:\, l\in P} x_P &= \sum_{j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{0j}} x_P \psi(P, l) \\
&= \frac{1}{4} \sum_{j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{0j}} x_P (\psi(P, l) + \psi(P, l_{\circlearrowleft}) \\
&\qquad + \psi(P, l_{\updownarrow}) + \psi(P, (l_{\circlearrowleft})_{\leftrightarrow})) \\
&= \frac{1}{4} \sum_{j\in\mathcal{N}} \sum_{P\in\mathscr{P}_{0j}} x_P |P| \\
&= \frac{1}{4} \sum_{j\in\mathcal{N}} d(0, j) \\
&= \frac{(N-1)\bar{L}}{4}.
\end{aligned}$$

The first equality is due to (40). The second in due to (44). The third one is due to (45). The fourth one is because $|P| = d(0, j)$ for each $P \in \mathscr{P}_{oj}$ and $\sum_{P\in\mathscr{P}_{0j}} x_P = 1$. The last equality follows from the definition of $\bar{L}$ and the symmetry of the network topology.

D. Proof of the Formula $\frac{j}{i} C_i$

Let node k receive packets from nodes $\{1, 2, \cdots, i\}$. The packets from first j nodes $\{1, 2, \cdots, j\}$ are for the receiver k. The rest of the packets are intended for other receivers, but node k happens to be in the neighborhood of each node in $\{j+1, j+2, \cdots, i\}$. The expected number of successful transmissions by the nodes $\{1, 2, \cdots, j\}$ is

$$\begin{aligned}
&\mathbb{E}\{\sum_{r=1}^{j} 1(\text{node } r \text{ is successful})\} \\
&\quad = \sum_{r=1}^{j} \Pr\{\text{node } r \text{ is successful}\} \\
&\quad = j \Pr\{\text{node } 1 \text{ is successful}\} \\
&\quad = j \tilde{C}_{i,1} \\
&\quad = j \sum_{m=1}^{i} \binom{i-1}{m-1} \frac{C_{i,m}}{\binom{i}{m}} \\
&\quad = j \sum_{m=1}^{i} \frac{m}{i} C_{i,m} \\
&\quad = \frac{j}{i} C_i.
\end{aligned}$$

E. Converse to Theorem 3

We adapt the notation in Part I: $W_{ij}(t)$ denotes the number of packets (with source i and destination j) successfully received by node j in slot t; $\mathcal{F}(t)$ is the set of all links with successful receptions in slot t.

Let λ be achievable with slotted ALOHA with transmission probability q. Every packet delivered from node i to j must be successfully transmitted $d(i, j)$ times. Therefore,

$$\sum_{t=0}^{T-1} \sum_{i,j\in\mathcal{N}} W_{ij}(t) d(i, j) \le \sum_{t=0}^{T-1} \sum_{l\in\mathcal{L}} 1(l \in \mathcal{F}(t)), \quad (46)$$

where the right hand side above is the total number of successful transmissions from slot zero to $T-1$. Observe the following chain:

$$\begin{aligned}
&N(N-1)\lambda\bar{L} \\
&\quad = \sum_{i,j\in\mathcal{N}} \lambda d(i, j) && (47) \\
&\quad \le \sum_{i,j\in\mathcal{N}} \mathbb{E}\{\liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} W_{ij}(t)\} d(i, j) && (48) \\
&\quad \le \sum_{i,j\in\mathcal{N}} \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}\{W_{ij}(t)\} d(i, j) && (49) \\
&\quad \le \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i,j\in\mathcal{N}} \mathbb{E}\{W_{ij}(t)\} d(i, j) && (50) \\
&\quad \le \liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{l\in\mathcal{L}} \Pr(l \in \mathcal{F}(t)) && (51) \\
&\quad = \sum_{l\in\mathcal{L}} \Pr(l \in \mathcal{F}(t)) && (52) \\
&\quad = 4N \Pr(l \in \mathcal{F}(t)) && (53) \\
&\quad = \frac{N}{4} \sum_{k=1}^{4} \binom{4}{k} q^k (1-q)^{5-k} C_k && (54) \\
&\quad \le N\bar{L}\, \eta_{ALOHA}. && (55)
\end{aligned}$$

(47) is the definition of $\bar{L}$; (48) follows from the definition of uniform achievability; (49) holds because of Fatou's lemma; (51) can be seen by taking expectation in (46); (53) is because there is total $4N$ (directed) links in a Manhattan network each with the same $\Pr(l \in \mathcal{F}(t))$; (54) is because of (20). The converse follows.

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