Stability, Capacity and Statistical Inference in Wireless Communication Networks

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Ph.D. defense talk, Nov. 12 2004

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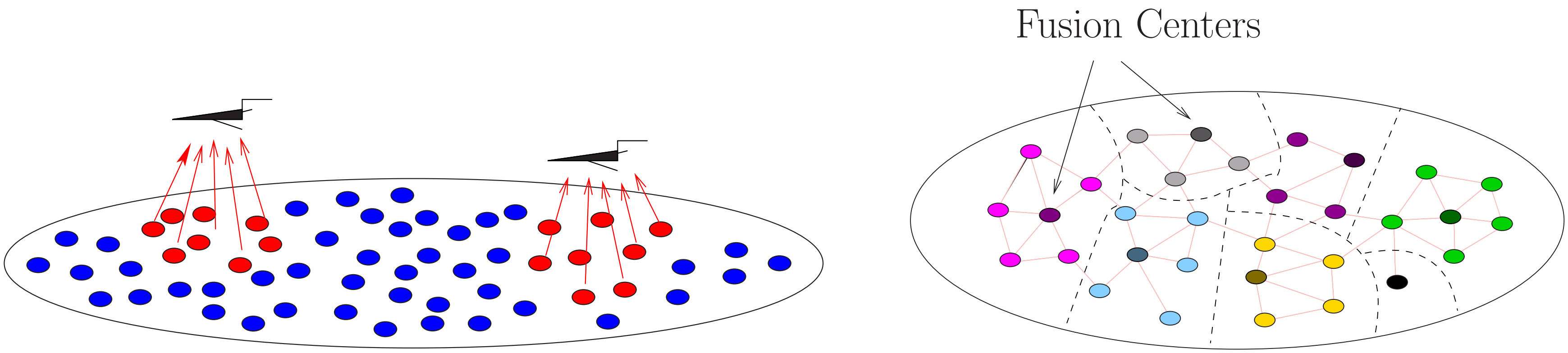
Thesis subjects

► Medium access and estimation for sensor networks

- Stability and capacity of networks with arbitrary topology

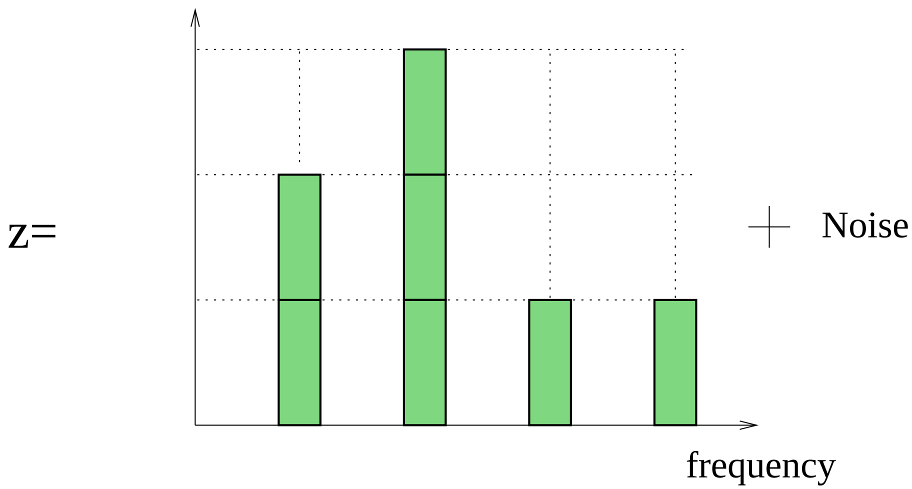
- Capacity of regular wireless networks

Sensor-Fusion Center Medium Access Problem



- Sensor networks are deployed to monitor an area or to detect targets.
- Sensor data are processed locally by fusion centers before delivery.
- How should the sensors communicate with the fusion center?

Type-Based Multiple Access (TBMA)



- Each sensor quantizes its data to finitely many levels.
- Transmits an orthogonal waveform (*e.g.*, a frequency tone) corresponding to its observation.

- The fusion center gets the addition of transmitted signals plus noise.
- Existence of target/parameter of interest is estimated from the received signal.

Performance of TBMA

- Estimation/detection performance of TBMA is analyzed* and compared with the approach that sensors transmit in non-overlapping time slots.

- The TBMA is shown to be asymptotically optimal in
  - Estimating a continuous parameter (performance metric= mean square estimation error).
  - Detecting a target (performance metric= error exponents).

- Performance under channel fading is analyzed.

$*$ G. Mergen and L. Tong "Estimation Over Multiaccess Channels Based on Types," submitted to *IEEE Transactions on Signal Processing*, July 2004 (revised in Oct.'04).

Outline

- Medium access and estimation for sensor networks

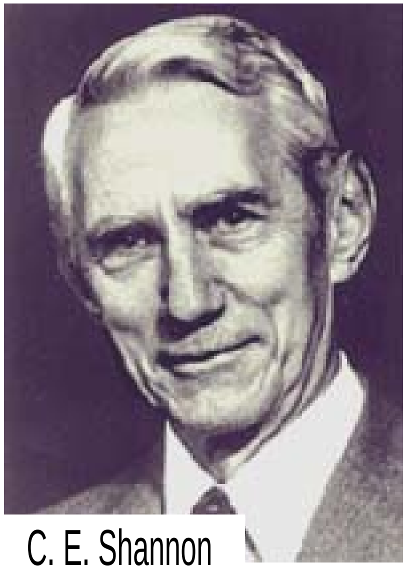
- ▶ Stability and capacity of networks with arbitrary topology
  - Network model
  - Characterization of capacity region
  - Transport capacity
  - Relations with buffer stability

- Capacity of regular wireless networks

Motivation

- Understanding the fundamental limits of communication systems is essential in assessing the performance of practical systems and aiding future designs.

- Shannon gave us the best example by developing the information theory of point-to-point channels.



C. E. Shannon

- However, the information theory of multiuser systems is extremely hard ($\exists$ as many open problems as you want).

- More recently, a new approch is emerging: develop and understand the fundamental limits of wireless networks from a networking perspective.

Related Work

- P. Gupta and P. R. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, 2000.

  ⇒ In a random network on a disc with N nodes, the max. per-node throughput scales as $O(1/\sqrt{N})$

- M. Grossglauser and D. Tse, "Mobility increases the capacity of ad hoc wireless networks", *INFOCOM* 2001.

  ⇒ If all nodes are mobile, the best per-node throughput is $O(1)$

- S. Toumpis and A. Goldsmith, "Capacity regions of ad hoc networks," *IEEE Trans. on Wireless Comm.*, 2003.

  ⇒ Numerically computed the capacity of "small" networks under varying channel conditions/different network protocols.

Motivation Cont'd

- Can we derive some guidelines for computing the capacity of wireless networks?
- Can we say anything beyond the order for large networks?
  - Yes, try networks with regular topology (grid, linear, etc.)
- Previous analyses assumed that there are infinitely many packets waiting at node buffers to be delivered. In reality, packets arrive randomly in time, and queue stability is an important issue.

  Does achievability of a certain throughput imply queue stability?

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A General Network Model

- Let $\mathcal{N} = \{1, \cdots, N\}$ denote the set of nodes in a network, and let $\mathcal{L} = \{(i,j) : i \neq j,\ i,j \in \mathcal{N}\}$ be the set of links.

- Nodes operate in synchronized slots. Each packet is one slot long.

- The reception channel of the network is specified by a set of conditional pdfs $\{\pi(\cdot;\mathcal{E}) :\ \mathcal{E} \subset \mathcal{L}\}$, where $\pi(\mathcal{F};\mathcal{E})$ is the conditional probability that the set of successful receptions is $\mathcal{F} \subset \mathcal{E}$ given that the set of transmissions is $\mathcal{E}$.

- This model takes into account:
  - Topological properties of the network (nodes pairs with shorter distance have higher reception probabilities in general)
  - Random channel fading (this is why the set of successful receptions is random).

A General Network Model Cont'd

- This model includes many previous models as special cases:
  - Collision channel model.
  - Multipacket reception (MPR) channel model (Ghez, Verdu, Schwartz '88)
  - Tassiulas and Ephremides '93.
  - Protocol model (Gupta and Kumar '00).
  - Asymmetric MPR model (Naware and Tong '03).

    ⋮

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Capacity Formulation

- It is assumed that every node has infinitely many packets waiting to be delivered to every other node in the network. Nodes are allowed to relay other nodes' packets.

Question: Given π, what set of communication rates are achievable in the network?

Definition: Suppose that node i wishes to communicate with node j with rate λ_{ij} [packets/slot]. The rate vector $(\lambda_{ij} : i, j \in \mathcal{N})$ is called achievable if there exists a scheduling policy that can guarantee those rates, $i.e.,$

$$\liminf_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} W_{ij}(t) \geq \lambda_{ij}, \quad \forall i, j \in \mathcal{N}$$

almost surely, where $W_{ij}(t)$ is the number of packets with source i and destination j delivered in slot t. The set of achievable rate vectors is called the capacity region of the network.

Randomized Time-Division (RTD) Policies

Routing: Let $\mathscr{P}_{ij}$ denote the set of all non-cyclic paths from node i to j. The routing in RTD is such that for every packet from i to j a random route $P \in \mathscr{P}_{ij}$ is assigned according to a certain probability distribution $\{x_P\}_{P \in \mathscr{P}_{ij}}$.

Medium Access: In each slot a random schedule $\mathcal{E}$ is chosen according to a probability distribution $\{p(\mathcal{E}\}_{\mathcal{E} \subset \mathcal{L}}$.

Queueing Discipline: If a node is scheduled to transmit over link l, a route $P \in \mathscr{P}_{ij}$ (such that $l \in \mathcal{P}$) is chosen randomly with probability proportional to $x_P \lambda_{ij}$, *i.e.*, each route passing through l is allocated bandwidth proportional to its traffic rate $x_P \lambda_{ij}$.

Remark: An RTD policy is specified by the probability distributions $\{p(\mathcal{E})\}_{\mathcal{E} \subset \mathcal{L}}$, $\{x_P\}_{P \in \mathscr{P}_{ij}}$, $\forall i \neq j$, and a set of target rates $(\lambda_{ij} : i, j \in \mathcal{N})$.

Characterization of Capacity Region

- Let $\Pi(l;\mathcal{E}) = \sum_{l \in F} \pi(\mathcal{F};\mathcal{E})$ be the marginal probability of success over link l.

Theorem 1:

A rate vector $(\lambda_{ij} : i, j \in \mathcal{N})$ is achieved by an RTD policy if

$$\sum_{i,j \in \mathcal{N}} \sum_{P \in \mathscr{P}_{ij} : l \in P} x_P \lambda_{ij} \leq \sum_{\mathcal{E} \subset \mathcal{L}} \Pi(l;\mathcal{E}) p(\mathcal{E}), \quad \forall l \in \mathcal{L}. \tag{1}$$

For given rate vector $(\lambda_{ij} : i, j \in \mathcal{N})$, if there does not exist an RTD policy satisfying (1), then this rate cannot be achieved by *any* policy. Hence, the capacity region is equal to the union of all $(\lambda_{ij} : i, j \in \mathcal{N})$ satisfying (1) with some RTD policy.

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An Upper Bound via Transport Capacity

- Computing the capacity region is formidable task. Sometimes simple upper bounds are desirable.
- Most networks have a natural distance metric.
- Let $d(i, j)$ denote the distance between the nodes i and j; equivalently, notation $d(l)$ will be used for $l = (i, j)$. The metric is assumed to satisfy the triangle inequality.

An Upper Bound via Transport Capacity Cont'd

Theorem 2:

All rate vectors $(\lambda_{i,j} : i, j \in \mathcal{N})$ in the capacity region satisfy

$$\sum_{i,j \in \mathcal{N}} \lambda_{ij} d(i,j) \leq \max_{\mathcal{E} \subset \mathcal{L}} \sum_{l \in \mathcal{L}} d(l) \Pi(l; \mathcal{E}). \quad (2)$$

- The left hand side can be viewed as the *work* (*i.e.*, rate-distance product) required to achieve the rate vector.
- We call the right hand side as the *transport capacity* of the network, *i.e.*, the maximum rate-distance product achieved by any schedule.
- Theorem provides an outer bound to the capacity region.

Network Capacity

Definition: Rate $\lambda \in \mathbb{R}$ is called *uniformly-achievable* if the rate vector $(\lambda_{ij} = \lambda : \ i, j \in \mathcal{N})$ is in the capacity region. The *network capacity* is defined as

$$\eta = (N-1)\max\{\lambda : \lambda \text{ is uniformly achievable}\}$$

Corollary of Theorem 2:

The network capacity satisfies

$$\eta \leq \frac{1}{\bar{L}N} \max_{\mathcal{E} \in \mathscr{E}} \sum_{l \in \mathcal{L}} d(l)\Pi(l; \mathcal{E}), \tag{3}$$

where $\bar{L}$ is the average distance between two arbitrarily selected nodes, *i.e.*,

$$\bar{L} = \frac{1}{N(N-1)} \sum_{i,j \in \mathcal{N}} d(i,j).$$

- This upper bound will be used extensively in the analysis of regular networks.

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Stability Formulation

- So far, it is assumed that nodes always have packets to be delivered. Consider the following alternative model.

- Suppose that packets with source i and destination j arrive at node i according to an ergodic and stationary process with rate λ_{ij}.

- Arrival processes for different i, j are assumed independent.

- Each node has an infinite buffer to hold packets, and the network starts operation with empty buffers.

Stability Formulation Cont'd

Question: Does there exists a scheduling policy stabilizing the node buffers for given arrival processes with rate vector $(\lambda_{ij} : i, j \in \mathcal{N})$?

Definition: Let $Q_i(t)$ be the number of packets in node i's buffer at time t. The buffer of node i is called *stable* if

$$\lim_{B \to \infty} \limsup_{t \to \infty} \Pr\{Q_i(t) > B\} = 0. \tag{5}$$

The network is called stable if all node buffers are stable; otherwise, it is unstable. The *stability region* is the closure of the set of arrival rates $(\lambda_{ij} : i, j \in \mathcal{N})$ for which the network can be stabilized with a scheduling policy.

Characterization of Stability Region

- Let $\Pi(\mathcal{F};\mathcal{E}) = \sum_{\mathcal{F}':\ \mathcal{F}\subset\mathcal{F}'} \pi(\mathcal{F};\mathcal{E})$ be the marginal probability of success over the link set $\mathcal{F}$.

- We assume that the channel satisfies

$$\Pi(\mathcal{F};\mathcal{E}') \leq \Pi(\mathcal{F};\mathcal{E}), \qquad \forall \mathcal{F} \subset \mathcal{E} \subset \mathcal{E}', \tag{6}$$

  $i.e.,$ the marginal probability of success decreases when there is more transmissions.

Theorem 3:

The network capacity and the stability regions are identical if the reception probabilities satisfy (6). In particular, let $(\lambda_{ij} : i, j \in \mathcal{N})$ be a rate vector satisfying the achievability condition (1) with strict inequalities, then the corresponding RTD policy stabilizes the network with arrival rate $(\lambda_{ij} : i, j \in \mathcal{N})$.

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  - ▸ Manhattan networks with scheduling
  - – Performance of suboptimal protocols
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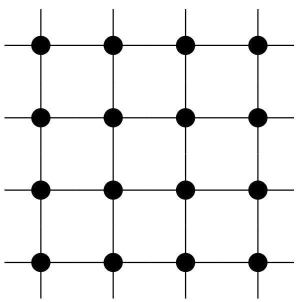
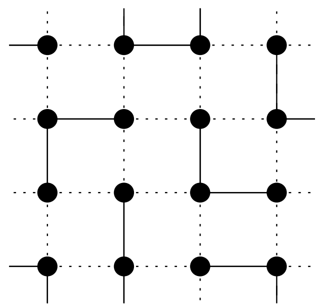
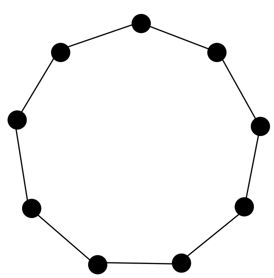
Regular Networks



a) Manhattan



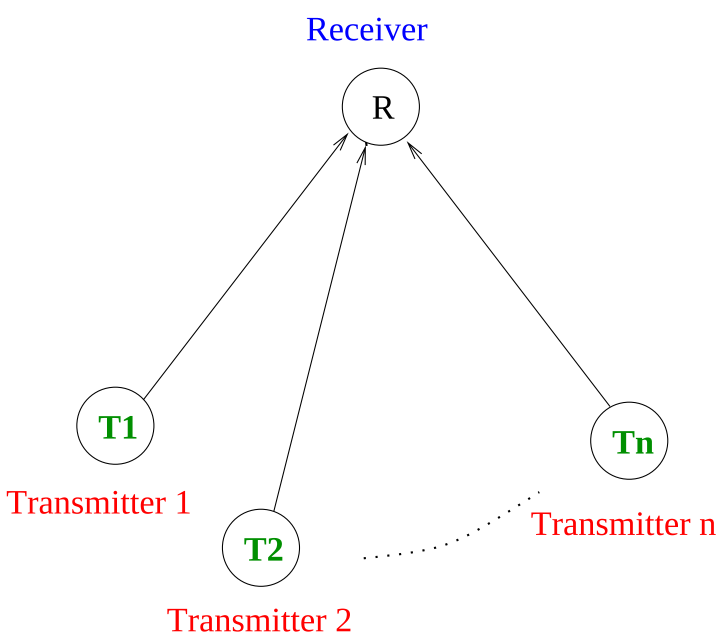
b) Manhattan
with fading links



c) Ring

- Topology is specified by a graph.
- Nodes are only allowed to communicate with neighbors.
- Nodes can transmit one packet at a time.
- A node cannot receive packets while transmitting.

Multipacket Reception (MPR) Model



- The reception probabilities are specified by an MPR matrix

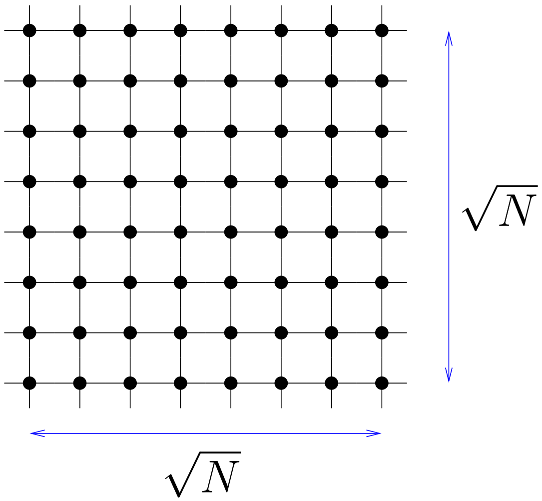
$$\mathbf{C} = \begin{pmatrix} C_{1,0} & C_{1,1} & & \\ C_{2,0} & C_{2,1} & C_{2,2} & \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

where

$$C_{n,k} = \Pr\{k \text{ packets are successfully received }; \; n \text{ packet are transmitted in the neighborhood}\}.$$

- The probability of success for any k group is $\frac{1}{\binom{n}{k}} C_{n,k}$.
- In a network, reception events at different receivers are independent given the set of transmitters.

Routing in Manhattan Networks



Lemma 1:

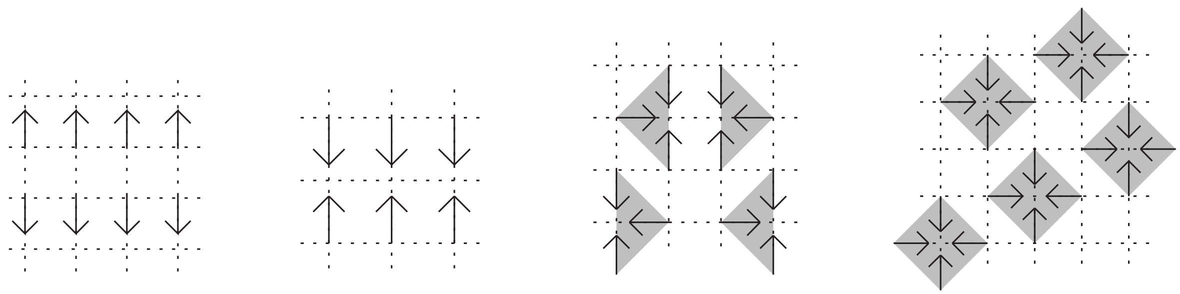
Consider the "lattice metric" (*i.e.,* the minimum number of hops to reach from one node to another). The average distance $\bar{L}$ between two arbitrary nodes is given by

$$\bar{L} = \begin{cases} \frac{\sqrt{N}}{2}, & \sqrt{N} \text{ odd} \\ \frac{N\sqrt{N}}{2(N-1)}, & \sqrt{N} \text{ even} \end{cases} = \frac{\sqrt{N}}{2} + O(1/\sqrt{N}). \qquad (8)$$

Lemma 2:

In a network with uniform arrival rate λ, all $\{x_P\}$ corresponding to a *symmetric, shortest path routing protocol* places $\frac{1}{4}\lambda(N-1)\bar{L}$ traffic over each directed link.

Transport Capacity of Manhattan Networks



a) 1−MPR
4x4 nodes

b) 2−MPR
3x3 nodes

c) 3−MPR
4x4 nodes

d) 4−MPR
5x5 nodes

Lemma 3: The transport capacity of Manhattan network is upper bounded by

$$\max_{i=1,\cdots,4} \frac{C_i}{i+1} N,$$

where $C_i = \sum_{j=1}^{i} jC_{i,j}$ is the expected of successful transmissions given i transmitters. The schedules above either achieve this transport capacity exactly or with a difference $O(\sqrt{N})$.

Remark: The proof of the upper bound relies on a linear programming relaxation of the optimization problem giving the transport capacity.

Capacity of Manhattan Networks

Theorem 4:

The capacity of Manhattan Network satisfies

$$\eta \;\le\; \max_{i=1,\cdots,4} \frac{C_i}{i+1}\frac{1}{\bar{L}}$$

$$\eta \;=\; \frac{1}{\sqrt{N}} \max_{i=1,\cdots,4} \frac{2C_i}{i+1} + O(\frac{1}{N}).$$

The schedule attaining the transport capacity achieves η. To assure fairness in medium access, shifted/rotated versions of the optimal schedules are used.

Remarks:

i) This theorem gives a more detailed description of the network capacity than just stating the order $O(1/\sqrt{N})$.

ii) The biggest advantage of regular network analysis is that we can find the *coefficient* besides the order. Many network functions (medium access, link fading, etc.) affect the coefficient but not the scaling.

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Slotted ALOHA Medium Access

- Topology-specific scheduling may not be implementable in practice, and it is important to quantify the capacity due to sub-optimal, yet more practical, control policies.
- In Slotted ALOHA medium access every node with a packet makes random transmission decisions, $i.e.,$ in the beginning of every slot it flips a coin, and decides to transmit with probability q.
- We consider the capacity setup, $i.e.,$ every node has infinitely many packets waiting to be delivered for every other node.

Network Capacity with Slotted ALOHA

Theorem 5:

The maximum uniformly achievable rate with slotted ALOHA is given by $\frac{\eta_{ALOHA}}{N-1}$, where

$$\eta_{ALOHA} = \frac{1}{4\bar{L}} \max_{0 \leq q \leq 1} \sum_{k=1}^{4} \binom{4}{k} q^k (1-q)^{5-k} C_k.$$

Remark

The proof uses a similar to Theorem 4:

i) Show that rates above η_{ALOHA} are not achievable by using the transport capacity.

ii) For every rate below, shortest path routing with the transmission probability q attaining the above maximum achieves η_{ALOHA}.

Random-Walk Based Routing

- In large networks, topology discovery may not be feasible, and nodes may not be able to shortest routes.
- Also, during network initialization nodes spend some time discovering the network and may not be able to use optimal routes.
- Flooding and random-walk based routing are alternatives that do not require nodes to know the network topology.
- In random-walk based routing, the packets are relayed at each consecutive hop to a randomly chosen neighbor with uniform probabilities.
- In a connected network, every packet eventually reaches its destination.

Network Capacity with Random-Walk Based Routing

Theorem 5:

The maximum uniformly achievable rate with random-walk based routing is

$$O(\frac{1}{N \log N}).$$

Remarks

i) This is significantly worse then $O(1/\sqrt{N})$.

ii) Our results suggest that the medium access method generally does not change the order of the capacity, but the routing does change the order, and a poor routing protocol can significantly degrade the performance of large networks.

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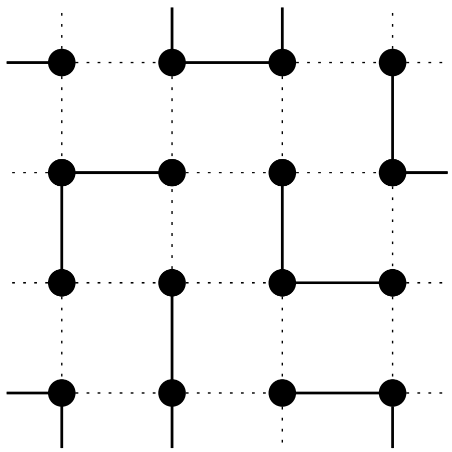
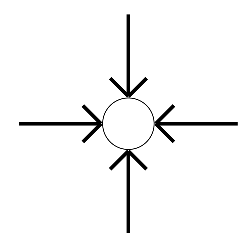
Manhattan Networks with Fading Links



- Each link of the Manhattan network is ON with probability p and OFF with probability $1-p$.
- The network controller *does not* know which links are ON or OFF.
- A reception is successful only if around the receiver there is a single transmitter whose link is ON.

The MPR matrix:

$$\mathbf{C}_p = \begin{pmatrix} 1-p & p & & & \\ 1-2p(1-p) & 2p(1-p) & 0 & & \\ 1-3p(1-p)^2 & 3p(1-p)^2 & 0 & 0 & \\ 1-4p(1-p)^3 & 4p(1-p)^3 & 0 & 0 & 0 \end{pmatrix}.$$



$C_{4,1} = 4p(1-p)^3$.

Capacity of Manhattan with Fading Links

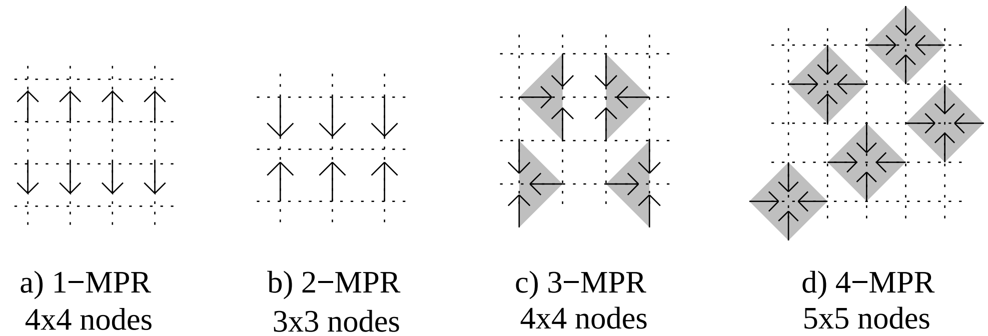
Network Capacity:

It follows from Theorem 4 that the capacity of the Manhattan network with fading links is

$$\eta = \frac{2p}{\sqrt{N}} \max_{i=1,\cdots,4} \frac{i(1-p)^{i-1}}{(i+1)} + O(\frac{1}{N}). \quad (9)$$

- Optimal schedule depends on the value of p:

$$i = \begin{cases} 1 & 1 \geq p \geq \frac{1}{4} \\ 2 & \frac{1}{4} \geq p \geq \frac{1}{9} \\ 3 & \frac{1}{9} \geq p \geq \frac{1}{16} \\ 4 & \frac{1}{16} \geq p \geq 0 \end{cases}$$



Capacity with Link State Information (LSI)

- Suppose the network controller knows the links which are ON or OFF.
- Denote the capacity with LSI by $\eta^{\#}$.

Theorem 6:

The following inequalities hold:

$$1 \leq \frac{\eta^{\#}}{\eta} \leq 2.86 + O(1/\sqrt{N}).$$

These inequalities are tight in two regimes:

$$\lim_{\substack{N \to \infty \\ p \to 0}} \frac{\eta^{\#}}{\eta} = 2.5 \quad \text{and} \quad \lim_{\substack{N \to \infty \\ p \to 1}} \frac{\eta^{\#}}{\eta} = 1. \tag{10}$$

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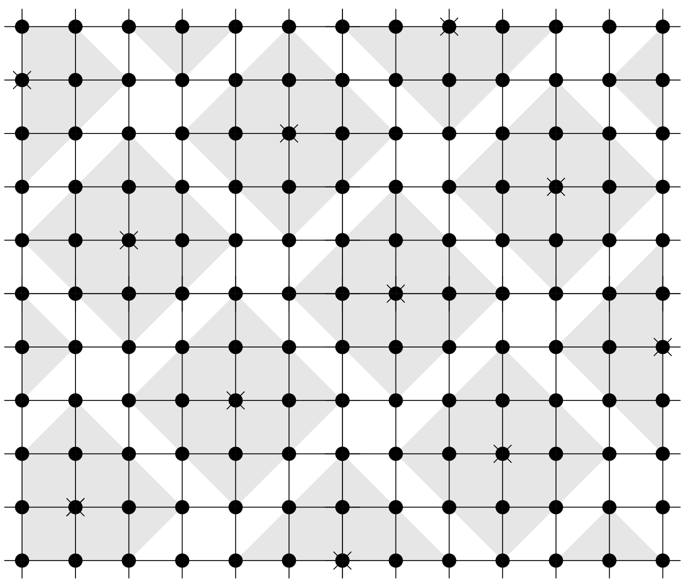
Capacity of Two-hop Connected Network

- Call a Manhattan network 2-hop connected if he nodes are connected to their two hop neighbors.

Observation:

- In a 2-hop connected Manhattan there exists a schedule which provides higher transport capacity:



- Works only under the condition that nodes can simultaneously receive 8 packets.
- The maximum transport in a 1-hop connected network is $\frac{4}{5}N + O(\sqrt{N})$.
- The neighboring schedule achieves $\frac{16}{13}N + O(\sqrt{N})$ transport.

Capacity of Two-hop Connected Network Cont'd

Theorem 7:

- The capacity of the 2-hop connected network with nodes capable of receiving 8 packets simultaneously satisfies

$$\frac{16}{13}\frac{1}{\bar{L}} + O(\frac{1}{N}) \leq \eta_{2-HOP}.$$

Remarks:

i) The capacity is at least 54% higher than the capacity of the 1-hop connected network.

ii) Several previous researchers have concluded that minimizing the connectivity radius while keeping the network connected leads to the highest throughput (*e.g.,* GK'00, GB'86).

iii) Our analysis points out that there potential benefits of non-minimal connectivity depending on network topology and channel usage (non-minimal connectivity is suboptimal for the ring topology as well).

Summary—Capacity Region of Arbitrary Networks

- We studied the capacity and stability problems in wireless networks.
- We first provided a general approach to characterizing the *capacity region* of wireless networks with a probabilistic reception model.
- This model includes several previous models as special cases, and is sufficiently general to include links with ergodic fading.
- A class of randomized policies is shown to achieve every rate inside the capacity region.
- Under a mild condition on the reception channel, the proposed policies are also shown to *stabilize the node buffers* for all arrival rates in the capacity region.
- We found a simple upper bound on the capacity region in terms of the *transport capacity*.

Summary—Regular Networks

- Previous works computed the scaling of network capacity under various assumptions.
- We analyzed several regular networks for which we could find the *coefficient* of the capacity besides the scaling law. The optimal medium access/routing policies are also found.
- We have seen that there are many factors that affect the coefficient, but not the scaling law.
- In particular, we explicitly characterized the effect of slotted ALOHA medium access, random-walk based routing, link fading and variable connectivity on the network performance.

The presented material will appear in: G. Mergen and L. Tong "Capacity and Stability of Regular Wireless Networks," accepted for publication in *IEEE Trans. Information Theory.*