Multipacket Reception with Multiple Antennas: A Cross-layer View

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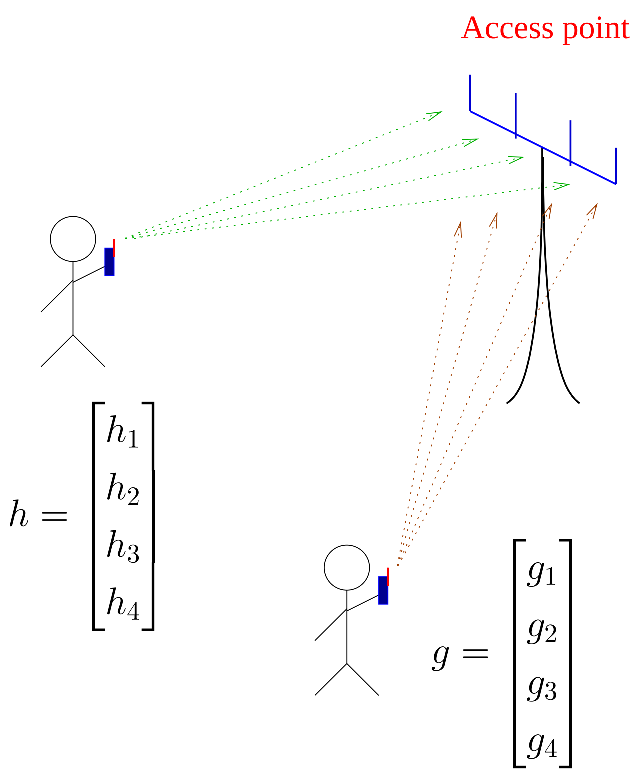
Motivation

- Last decade has seen a tremendous improvement in physical layer of wireless networks.
- Multiple antennas and advanced signal processing techniques continue to push capacity limits further.
- But what are the effects of an advanced physical layer to higher layers of networks? In particular, how is the medium access (MAC) layer affected?

Multiple Antennas

- Multiple antennas have been shown to provide diversity and improve communication rates.
- Besides, they can increase the range of communication.
- A less investigated capability is the possibility of multipacket receptions.

Multiple Antennas Cont'd



- Suppose each node has a single antenna except the access point.
- Multiple users can be served simultaneously if their spatial signatures are orthogonal, or close to orthogonal.
- However, the MAC layer in many wireless systems is designed to serve one user at a time (*e.g.*, IEEE 802.11, GSM standard).

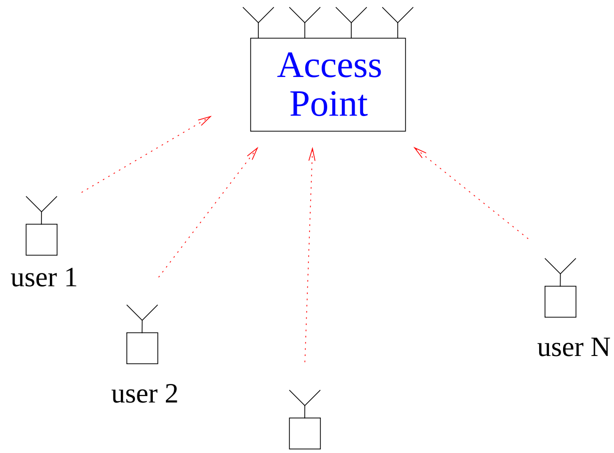
Objective

- Improve bandwidth utilization and total throughput by exploiting multipacket receptions (MPRs).
- Establish some guidelines and design principles for systems with MPR.
- Understand the effect of MPR on network throughput.

Outline

- Introduction
- Up-link Medium Access with MPR
  - ▶ Multipacket Reception Model
  - – Capacity Region
  - – Stability Region of Slotted ALOHA
  - – Multiple Antenna Network
- Summary and Extensions

Multipacket Reception (MPR) Model

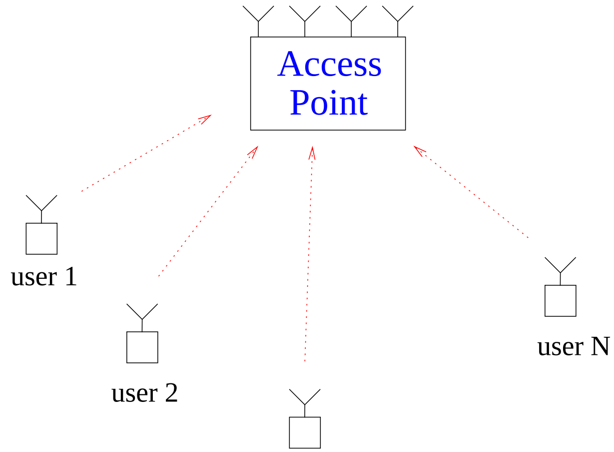


- Consider a slotted system.
- Let $\mathcal{U} = \{1, 2, \cdots, N\}$ be the set of users.
- The MPR channel is specified by the reception probabilities

$$Q_{\mathcal{R}|\mathcal{T}} = \Pr\{\text{Users in } \mathcal{R} \text{ are received} \mid \mathcal{T} \text{ transmitted}\},$$

where $\mathcal{R} \subset \mathcal{T} \subset \mathcal{U}$.

MPR Model Cont'd



- For $N = 2$, the model is specified by

$$\{Q_{1|1},\ Q_{2|2},\ Q_{1|1,2},\ Q_{2|1,2},\ Q_{1,2|1,2}\}.$$

- For the extreme case of collision channel, these probabilities are

$$\{1, 1, 0, 0, 0\}.$$

- In general these probabilities can take values between 0 and 1 depending on
  - Existence of channel fading
  - Number of antennas
  - Specifics of the multiuser receiver.

Our Approach

- Determine the performance of a network with *general* $\{Q_{\mathcal{R}|\mathcal{T}}\}$.
- By substituting specific $\{Q_{\mathcal{R}|\mathcal{T}}\}$ corresponding to the situation of interest, one can analyze the performance of different networks.
- Consider the previously mentioned multiple antenna network to see how the network performance is affected by various physical layer parameters.

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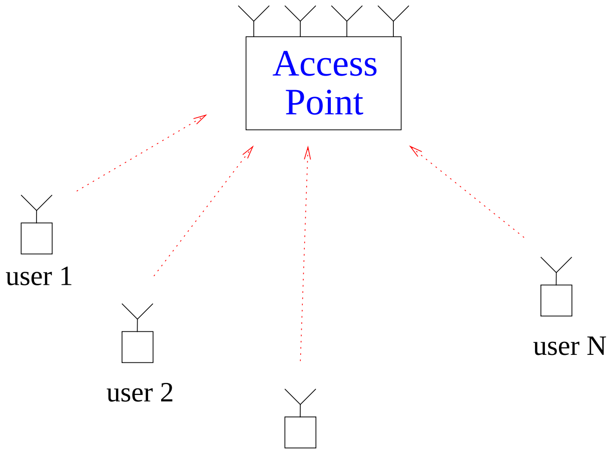
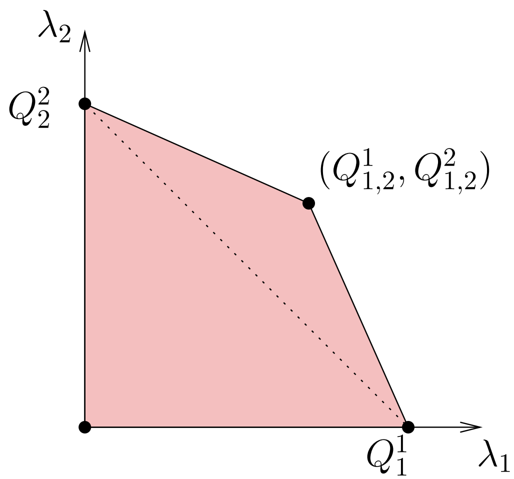
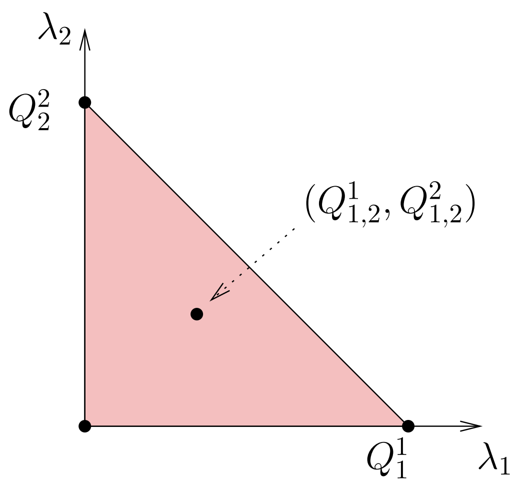
Capacity Region



- Suppose that every node has infinitely many packets waiting to be transmitted.
- The access point can schedule at will.
- Given $\{Q_{\mathcal{R}|\mathcal{T}}\}$, what rate vectors $(\lambda_1, \cdots, \lambda_N)$ [packets/slot] are achievable in the long run?

Capacity Region Cont'd





TDMA achievable rates for N=2

Observations for N=2:

- If the *first* user is scheduled all the time, the achieved rate is $Q_1^1 := Q_{1|1}$.
- If the *second* user is scheduled all the time, the rate $Q_2^2 := Q_{2|2}$ is achieved .
- If both user are scheduled all the time, the rate vector $(Q_{1,2}^1, Q_{1,2}^2)$ is achieved, where

$$Q_{1,2}^1 := Q_{1|1,2} + Q_{1,2|1,2}, \qquad Q_{1,2}^2 := Q_{2|1,2} + Q_{1,2|1,2}.$$

  are individual success probabilities.
- Time-sharing among different schedules gives the convex hull.

Capacity Region Cont'd

- For general N, let $Q^i_{\mathcal{T}} = \sum_{\mathcal{R}:\ i \in \mathcal{R}} Q_{\mathcal{R}|\mathcal{T}}$ be the success probability of user i with schedule $\mathcal{T}$.

Theorem 1: (Capacity Region)†

- The time-sharing policy achieves every rate in the set

$$\mathcal{C} = \text{convex hull}\{(Q^1_{\mathcal{T}}, \cdots, Q^N_{\mathcal{T}}) : \mathcal{T} \subset \mathcal{U}\}.$$

No scheduling policy can achieve rates outside $\mathcal{C}$.

Remark:

- The theorem provides an outer bound to the throughput region of any MAC protocol.

† [1] G. Mergen and L. Tong "Capacity and Stability of Regular Wireless Networks," to appear in *IEEE Trans. Information Theory*.

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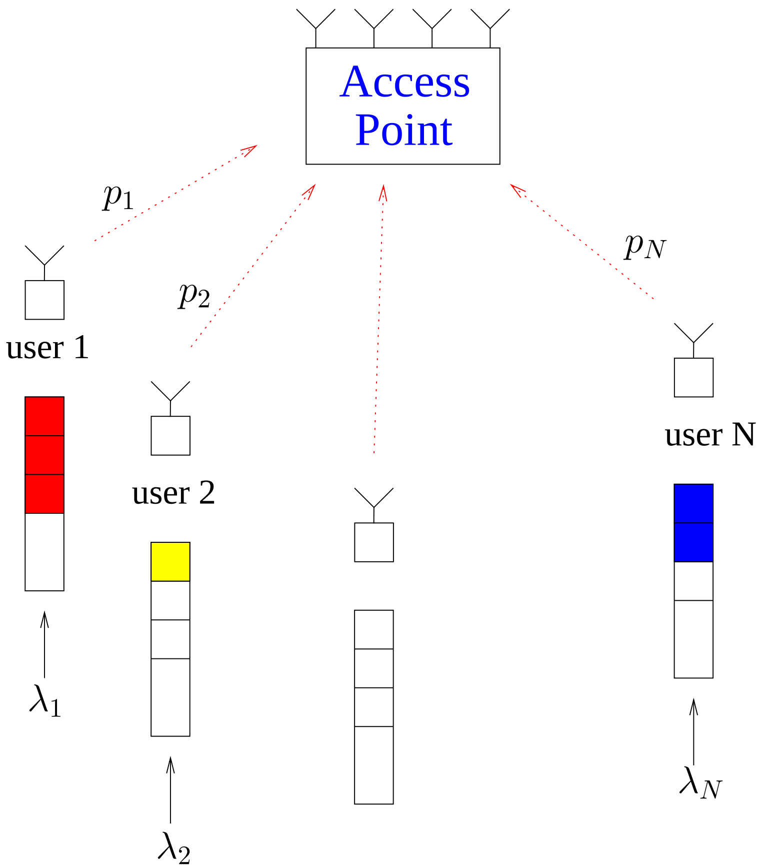
Slotted ALOHA

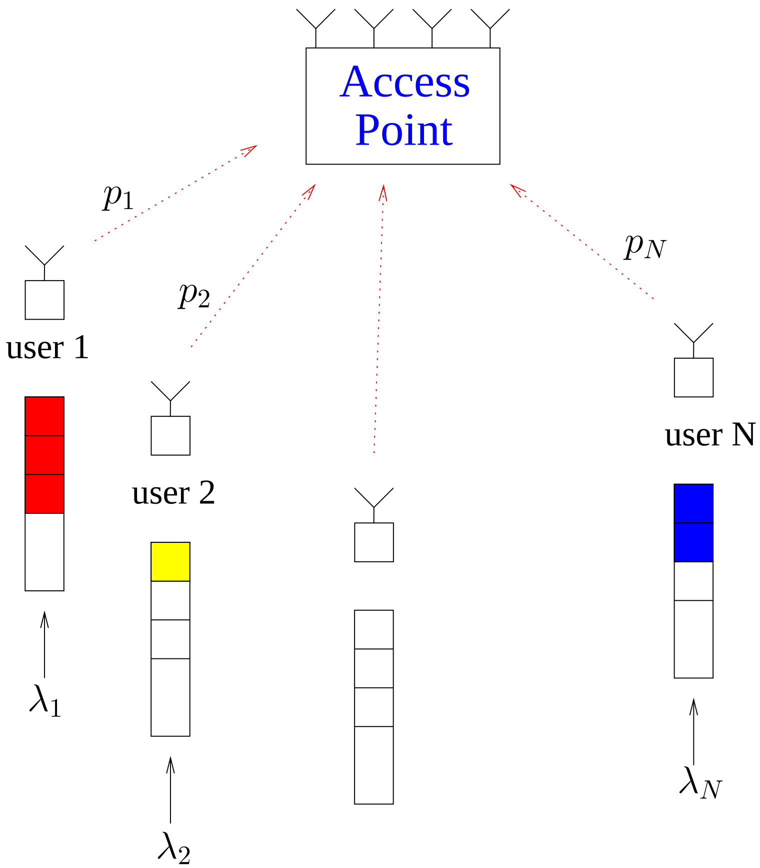
- Time-sharing among different schedules requires control and coordination of the access point.
- In practice, distributed protocols such as slotted ALOHA are more desirable, since they require less coordination.
- How does the throughput of slotted ALOHA compare with the capacity region?

Slotted ALOHA Cont'd



- Each user has an infinite buffer to hold his/her packets.
- Packets arrive randomly to user i with rate λ_i according to an i.i.d. stochastic process.
- If the buffer is non-empty, a packet is transmitted with probability p_i.
- The successful transmissions are determined randomly according to $\{Q_{\mathcal{R}|\mathcal{T}}\}$.

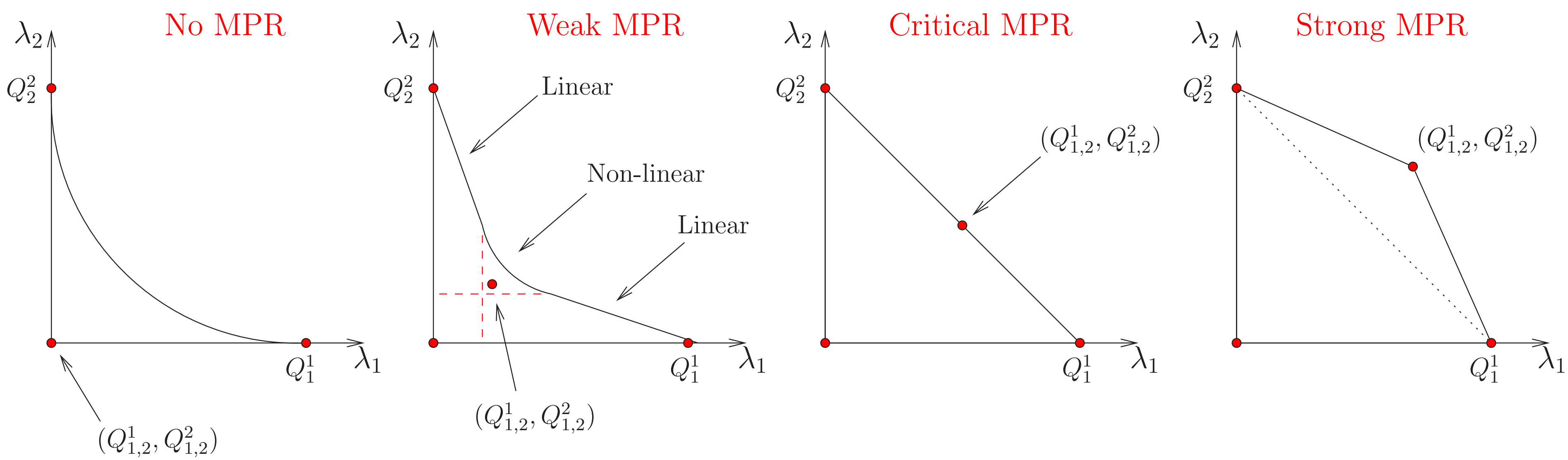
Slotted ALOHA Cont'd



Definitions:

- A network is called stable if the queue sizes remain bounded (*i.e.*, they don't go to infinity).
- Stability region of slotted ALOHA is the union of all arrival rates $(\lambda_1, \cdots, \lambda_N)$ such that the network is stable with some $(p_1, \cdots, p_N)$.

Stability Region of ALOHA



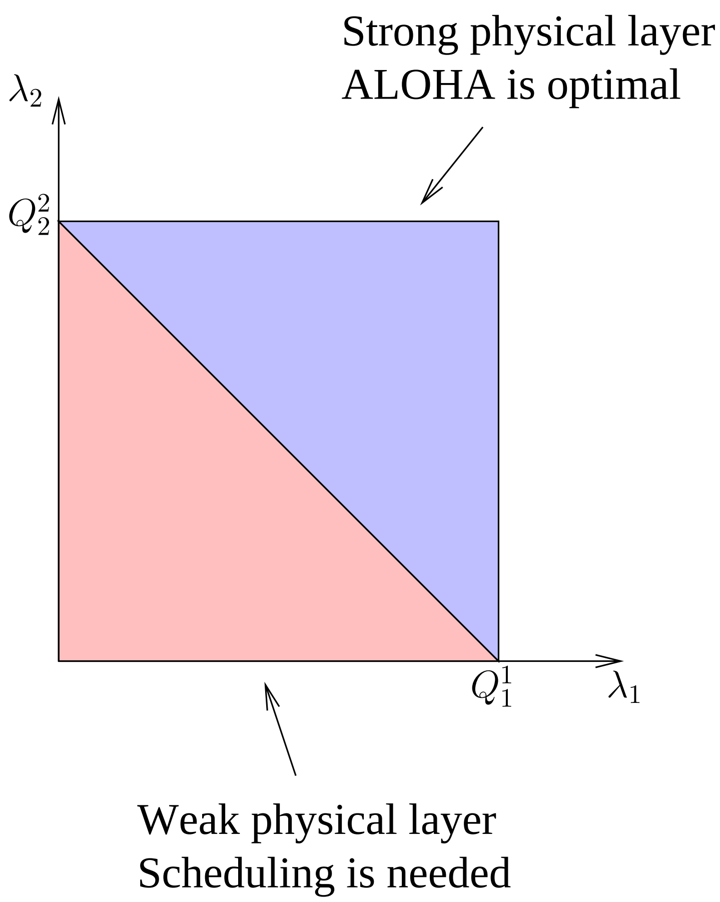
Theorem 2:

For $N = 2$, the stability region of slotted ALOHA has one of the above forms (an explicit characterization is given in [2][‡]). In particular, the ALOHA stability region coincides with the capacity region if

$$\frac{Q_{1,2}^1}{Q_1^1} + \frac{Q_{1,2}^2}{Q_2^2} \geq 1. \qquad (1)$$

‡[2] V. Naware, G. Mergen and L. Tong, "Stability and Delay of Slotted ALOHA with Multipacket Reception," to appear in *IEEE Trans. Information Theory.*

Stability Region of ALOHA Cont'd



- The condition for ALOHA optimality (eqn. (1)) is another way of saying that the point $(Q_{1,2}^1, Q_{1,2}^2)$ lies above the line joining Q_1^1 and Q_2^2.
- The theorem indicates that if the physical layer is strong enough, scheduling is not needed; a simple protocol such as slotted ALOHA is optimal.

What's next?

- The theory so far was for general $\{Q_{\mathcal{R}|\mathcal{T}}\}$.
- Next, we will consider specific $\{Q_{\mathcal{R}|\mathcal{T}}\}$ corresponding to a multiple antenna network, and see how different receivers and other physical layer parameters affect the stability and capacity regions.

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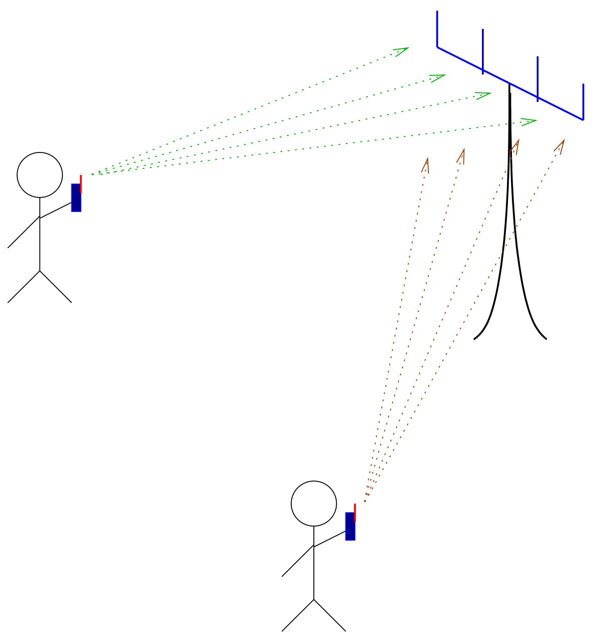
Physical Layer Model



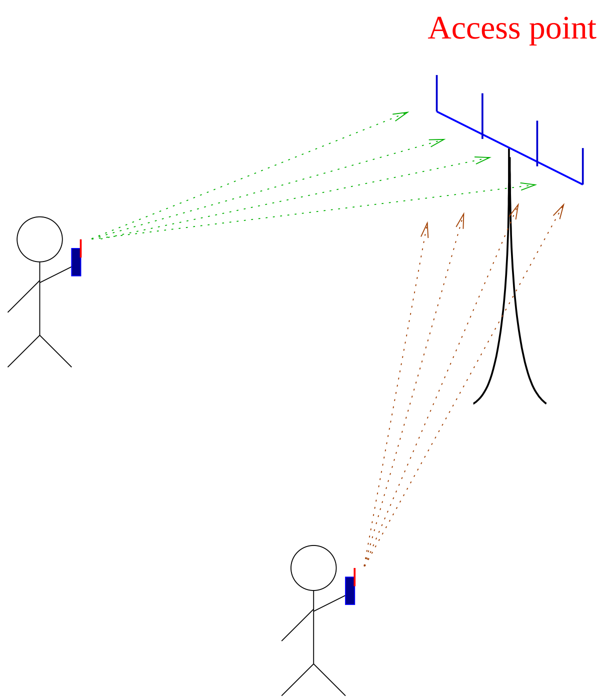
- Consider a two user system.
- M receive antennas as a linear array.
- Most of the energy from user i transmission is received from a planar wavefront arriving at angle θ_i.
- Rayleigh distributed channel gain: $h_i \sim \mathcal{CN}(0, \sigma_i^2)$.
- The received signal is $\mathbf{y} = \mathbf{VHs} + \mathbf{n}$, where

$$\mathbf{V} = [\mathbf{v}_1 \;\; \mathbf{v}_2], \quad \mathbf{v}_i = [1 \;\; e^{j\theta_i} \;\; \cdots \;\; e^{j(M-1)\theta_i}]^T.$$

$$\mathbf{H} = \text{diag}(h_1, h_2), \quad \mathbf{n} \sim \mathcal{CN}(0, \mathbf{I}_m)$$

$$\mathbf{s} = [s_1 \;\; s_2]^T \text{ (users' tx'ed symbols)}, \; \mathbb{E}(|s_i|^2) = 1.$$

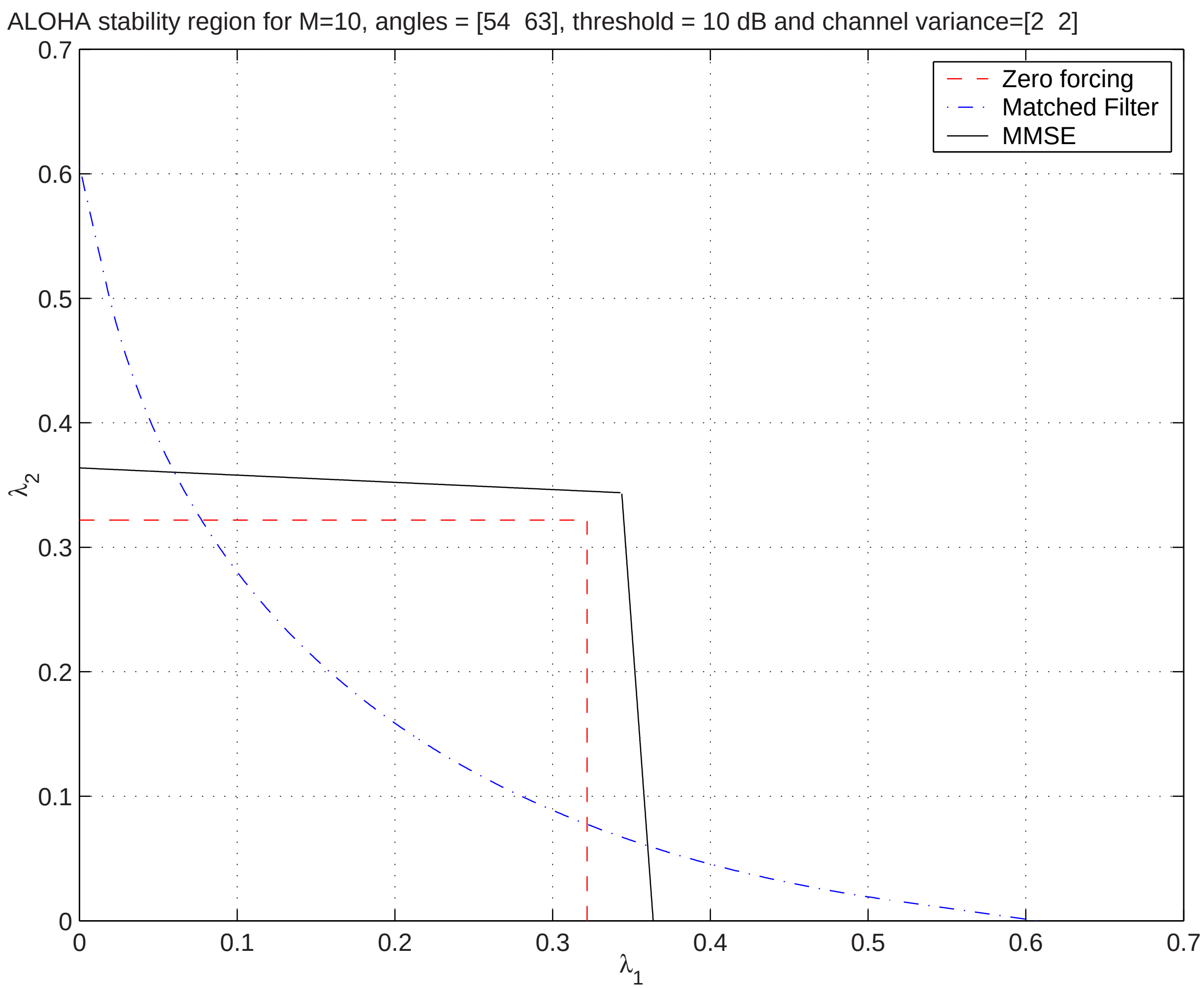
Physical Layer Model Cont'd



Receiver:

- The receiver is assumed to know $\mathbf{VH}$.
- Receiver processing is linear, *i.e.*, multiply $\mathbf{y}$ with some matrix $\mathbf{F}$ to distinguish users.
- Three types of receivers are considered
  - Matched Filter
  - Decorrelater (zero-forcing)
  - MMSE.
- Transmission is successful if SINR$\geq$Threshold.
- Reception probabilities $\{Q_{\mathcal{R}|\mathcal{T}}\}$ can be computed numerically.

Quantitative Insights

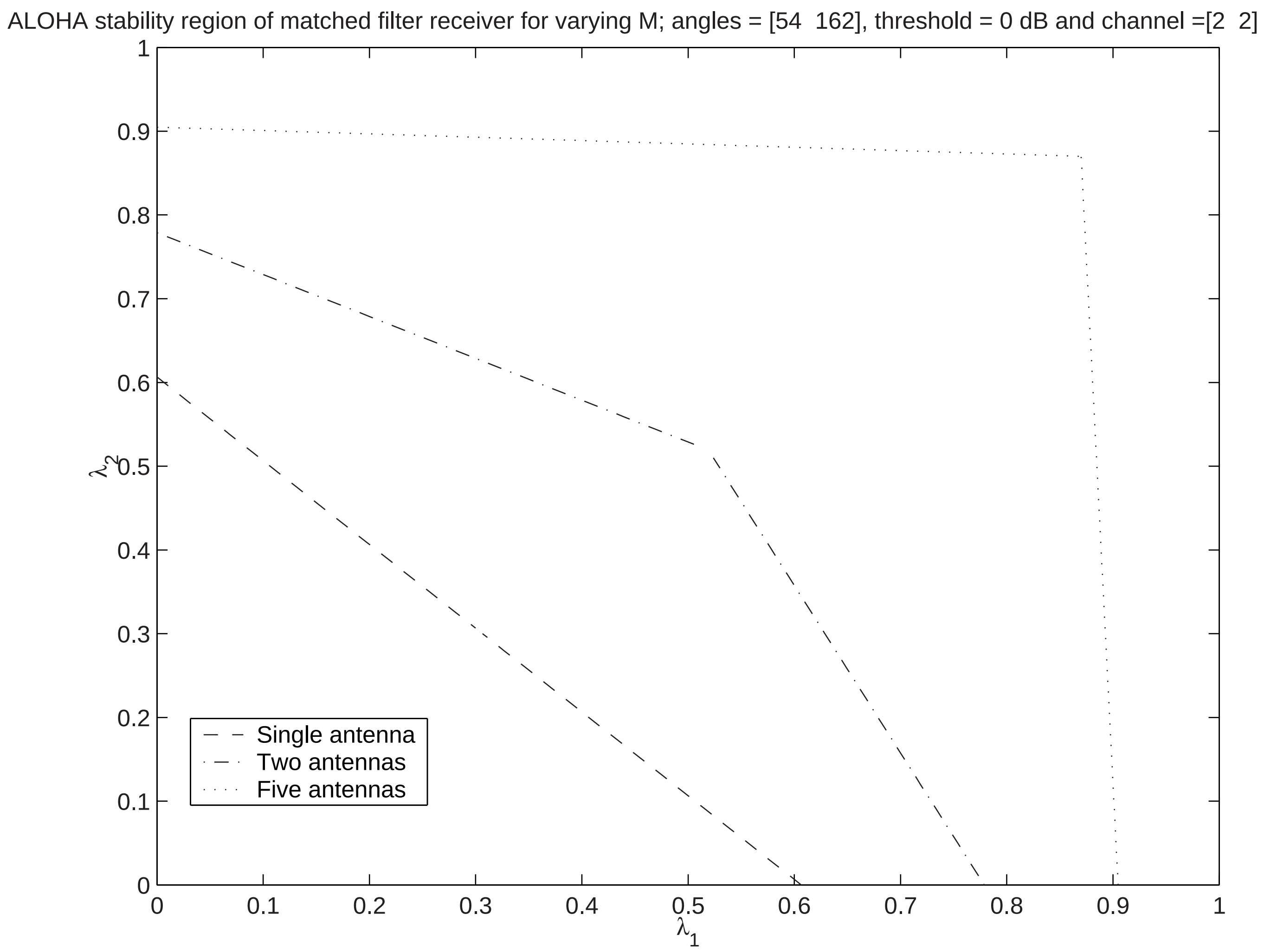


Quantitative Insights Cont'd

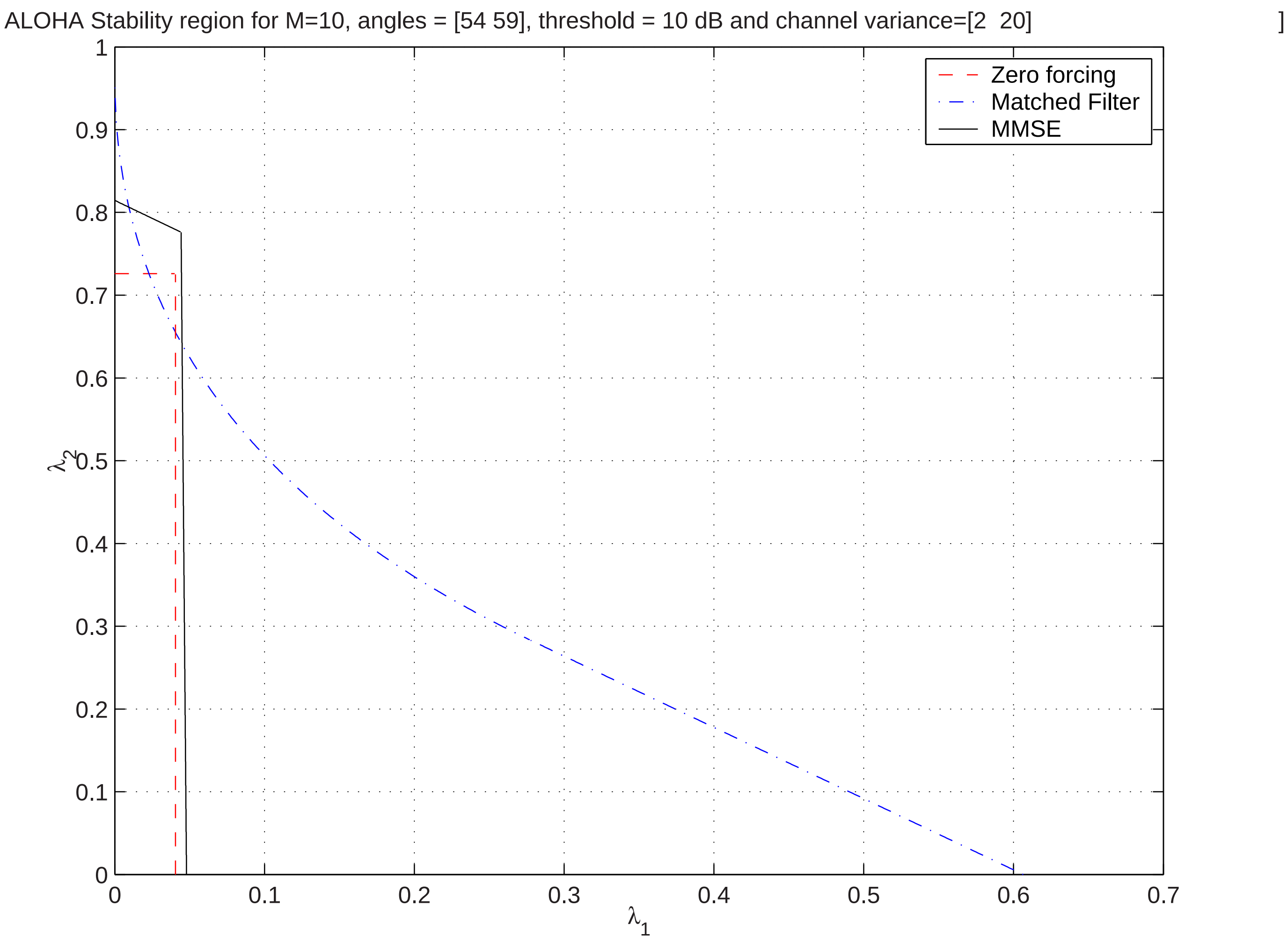
Remarks:

- The MMSE and zero-forcing receivers are better when both users transmit together.
- Along the λ_1, λ_2 axes they are worse since they don't distinguish whether both users are there or not.
- When the receiver uses the MMSE or zero-forcing receivers, slotted ALOHA is optimal and scheduling is not needed.

Varying Number of Antennas



Asymmetric Channel Gains



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Summary and Extensions

- With the use of multipacket reception (MPR) the same time slot can be used by multiple users simultaneously.
- The MPR property can be exploited to improve the system throughput.
- We developed a framework to analyze MPR systems.
- This framework uses the notion of "capacity region" as a benchmark to assess the performance of
  - Receiver front-ends.
  - Medium access/scheduling protocols.
- In particular, we analyzed the performance of the slotted ALOHA protocol, and showed that it is throughput-optimal if the physical layer is strong enough.

Summary and Extensions Cont'd

- Our analysis of the multiple antenna system was brief, yet more systems can be analyzed once the reception probabilities are specified.
- Other results which I didn't have time to talk about are
  - Extensions to more than two users [1,2].
  - Closed form expressions for queuing delay with slotted ALOHA [2]. ALOHA is shown to be delay-optimal under condition (1).
  - Network stability with scheduling [1].
  - Capacity of regular ad hoc networks with MPR [1].

References

[1] G. Mergen and L. Tong “Stability and Capacity of Regular Wireless Networks ,” to appear in *IEEE Trans. Information Theory*.

[2] V. Naware, G. Mergen and L. Tong, “Stability and Delay of Slotted ALOHA with Multipacket Reception,” to appear in *IEEE Trans. Information Theory*.